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Modeling and Solving Fractional Differential Algebraic Equations in Smart Control Systems

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Abstract. *In this have a look at, a new mathematical model for FDAE-based smart manage systems is proposed. The model carries fractional derivatives blended with algebraic constraints to symbolize prolonged memory results. We describe a numerical method to solve the proposed device and practice this version to robotics, self-reliant cars, and sensible prosthetics. The Fractional Collocation Method is employed to resolve FDAEs, making sure accuracy and balance. To validate the proposed method, we introduce 3 examples: a simple FDAE demonstrating the accuracy of the numerical solution, a device of FDAEs modeling interdependent dynamic variables with algebraic constraints, and an FDAE with a nonlinear algebraic constraint, highlighting the approach's capability to handle complicated, nonlinear dynamics. Simulation results verify that FDAEs offer a more practical and powerful tool for designing and reading wise manage systems as compared to classical techniques.*

Keywords: *Fractional Differential Algebraic; Numerical approach; Smart Control Systems*

Abstrak. Dalam kajian ini, diusulkan model matematika baru untuk sistem manajemen cerdas berbasis FDAE. Model tersebut memuat turunan fraksional yang dicampur dengan batasan aljabar untuk melambangkan hasil memori yang diperpanjang. Kami menjelaskan metode numerik untuk menyelesaikan perangkat yang diusulkan dan menerapkan versi ini pada robotika, mobil mandiri, dan prostetik cerdas. Metode Kolokasi Fraksional digunakan untuk menyelesaikan FDAE, memastikan akurasi dan keseimbangan. Untuk memvalidasi metode yang diusulkan, kami memperkenalkan 3 contoh: FDAE sederhana yang menunjukkan akurasi solusi numerik, perangkat FDAE yang memodelkan variabel dinamis yang saling bergantung dengan batasan aljabar, dan FDAE dengan batasan aljabar nonlinier, yang menyoroti kemampuan pendekatan untuk menangani dinamika nonlinier yang rumit. Hasil simulasi memverifikasi bahwa FDAE menawarkan alat yang lebih praktis dan kuat untuk merancang dan membaca sistem manajemen yang bijaksana dibandingkan dengan teknik klasik.

Kata kunci: Aljabar Diferensial Fraksional; Pendekatan numerik; Sistem Kontrol Cerdas

1. INTRODUCTION

We consider the fractional differential algebraic equations (FDAEs) on the form

$$\begin{gathered}\mathcal{D}^{\alpha}\mathcal{X}(t)=\mathcal{F}\big(t,\mathcal{X}(t),\mathcal{U}(t)\big)\\ \mathcal{G}\big(t,\mathcal{X}(t)\big)=0\end{gathered}\tag{1}$$

FDAEs (1) combine fractional differential equations ($D^{\alpha}\,\mathcal{X}(t)$) with algebraic constraints $\mathcal{X}$ (t) is the state vector, f represents the dynamics, $\mathcal{U}(t)$ is the manipulate input, and g represents the algebraic constraints[2,6,10]. Fractional Differential Algebraic Equations (FDAEs) are a effective device for modeling structures together with robotics, advanced prosthetics , and independent motors. Since FDAEs combining algebraic constraints with fractional derivatives, so is taking into consideration a more correct description of dynamic behavior[1-9]. These systems offer a greater accurate mathematical illustration of structures which can be stricken by their behavior, which include the consequences of friction in robotics or time delays on top of things structures[2-6].

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Many researchers have studied fractional derivatives and their applications in complex dynamical structures. Sherry and Paliano (2019) studied mathematical fashions primarily based on FDAEs and validated their effectiveness in modeling dynamical systems with algebraic constraints [1-2,8], Podlubny (1998) offered a complete evaluation of fractional differential equations and their applications in physics and engineering [5,7,10]. Atangana and Paliano (2016) proposed a brand new definition of fractional derivatives the use of a non-singular kernel, which stepped forward the stableness of numerical answers of fractional equations [4].

This paper ambitions to give a new mathematical model primarily based on FDAEs for utility in clever manage structures. The Fractional Collocation Method is proposed as an powerful numerical technique to resolve these equations. The model is evaluated thru 3 case research, consisting of a easy FDAE, a device with algebraic constraints, and a extra complicated case with nonlinear constraints. Simulation effects display that the proposed approach provides higher accuracy and balance as compared to standard fashions, making it a useful device within the design of advanced wise structures.

2. Preliminaries

This section contains the important basic definitions we need to study FDAEs. Fractional derivatives generalize classical derivatives to non-integer orders. The most common definitions include

Definition 2.1[7] A Liouville-Caputo derivative of system (1) define as

$$\mathcal{D}^{\alpha}\mathcal{X}(t)=\frac{1}{\Gamma(1-\alpha)}\int_{0}^{t}\frac{\mathcal{X}'(\mathcal{T},\mathcal{X}(\mathcal{T}),\mathcal{U}(\mathcal{T}))}{(t-\mathcal{T})^{\alpha}}d\mathcal{T},\ \alpha\in(0,1)$$

where Γ is the Gamma function.

Definition 2.2 [7] A Caputo-Fabrizio derivative of system (1) is given by

$$\mathcal{D}^{\alpha}\mathcal{X}(t)=\frac{1}{1-\alpha}\int_{0}^{t}\mathcal{X}'\left(\mathcal{T},\mathcal{X}(\mathcal{T}),\mathcal{U}(\mathcal{T})\right)\exp\left(-\frac{\alpha}{1-\alpha}(t-\mathcal{T})\right)d\mathcal{T}$$

this definition avoids singularities in the kernel.

Definition 2.2 A fractional derivatives in $\mathcal{U}(\mathcal{T})$define as

$$\mathbb{F}_{\text{friction}}=\mathcal{m}\mathcal{D}^{\alpha}\mathcal{V}(\mathcal{T})$$

Where $\mathcal{V}(\mathcal{T})$ is the velocity vector, $\mathcal{m}$ is the friction coefficient.

This definition showing important fractional derivatives are used to model long memory effects, for example friction in robotic systems.

3. Some Important Theorems

In this section, we present the important and basic theorems used in the study of PDAEs. The theorem of the existence of oneness of solutions, the theorem of convergence, and the study of stability are presented. Here are the theorems in below**.**

Theorem.3.1: The system (1) has index-1 if the matrix $\mathcal{G}_{\mathrm{x}} \neq 0$ on interval $[0, T]$.

Proof

By differentiating the algebraic constraint respect to variable t, we get

$$\frac{d}{dt}\mathcal{G}\big(t, \mathcal{X}(t)\big) = \mathcal{G}_{\mathrm{t}} + \mathcal{G}_{\mathrm{x}}\, \mathcal{D}^{\alpha}\mathcal{X}(t) = 0$$

From the system (1) we have

$$\mathcal{G}_{\mathrm{t}} + \mathcal{G}_{\mathrm{x}}\mathcal{F}(t, \mathcal{X}(t), \mathcal{U}(t)) = 0$$

Sequently,

$$\mathcal{G}_{\mathrm{x}} = -\mathcal{G}_{\mathrm{t}}\left(\mathcal{F}\big(t, \mathcal{X}(t), \mathcal{U}(t)\big)\right)^{-1}$$

Then $\mathcal{G}_{\mathrm{x}}$ is invertible, and the system (1) can be solved explicitly after one differentiation. Hence, the system of index-1.

Theorem.3.2: Let $\mathbb{v}(\mathcal{X})$ a Lyapunov function in [4], if there exists $\mathbb{v}(\mathcal{X})$ satisfying $\mathcal{D}^{\alpha}\mathbb{v}(\mathcal{X}) \leq 0$, then the system (1) is stable.

Proof

Using definition 2.2 obtain as

$$\mathcal{D}^{\alpha}\mathbb{v}(x) = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{\mathbb{V}'\big(\mathcal{X}(\mathcal{T})\big)}{(t-\mathcal{T})^{\alpha}}\, d\mathcal{T}.$$

Then if $\mathbb{V}'(\mathcal{X}(\mathcal{T})) \leq 0$, then $\mathcal{D}^{\alpha}\mathbb{v}(x) \leq 0$. Therefore, By Lyapunov's theorem, the system (1) is stable.

Theorem .3.3: Let $\mathcal{D}^{\alpha}\mathcal{X}(t)$ is Lipschitz continuous in $\mathcal{X}$ and $\mathcal{U}$, and the algebraic constraint is continuously differentiable with $\mathcal{G}_x \neq 0$, then the FDAE system (1) has a unique solution for $\mathcal{X}(t)$ and $\mathcal{U}(t)$.

Proof

Since the part of system (1) $\mathcal{F}(t, \mathcal{X}(t), \mathcal{U}(t))$ is satisfying Lipschitz continuous in $\mathcal{X}, \mathcal{U}$. Then $\exists\ \mathfrak{m}_1, \mathfrak{m}_2$ are constants; $\forall\ \mathcal{X}_1, \mathcal{X}_2, \mathcal{U}_1, \mathcal{U}_2$, such that

$$\|\mathcal{F}(t, \mathcal{X}_1, \mathcal{U}_2) - \mathcal{F}(t, \mathcal{X}_1, \mathcal{U}_2)\| \leq \mathfrak{m}_1\|\mathcal{X}_1 - \mathcal{X}_2\| + \mathfrak{m}_2\|\mathcal{U}_1 - \mathcal{U}_2\|.$$

The part two of (1) $\mathcal{G}(t, \mathcal{X}(t)) = 0$ is continuously differentiable, and its Jacobian $\frac{\partial \mathrm{g}}{\partial \mathrm{x}}$ is invertible.. Then we have $\mathcal{G}x$ is nonsingular $\forall\ t$ such that $0 \leq t \leq T$.

Now, we can solve $\mathcal{G}(t, \mathcal{X}(t)) = 0$ uniquely for $\mathrm{x}(t)$ in $\mathrm{u}(t)$ and t. By the Implicit Function Theorem, there exists a unique function $\mathbb{H}(t, \mathcal{U}(t)$ satisfying

$$\mathcal{G}(t, \mathbb{H}(t, \mathcal{U}(t))) = 0$$

where $\mathcal{X}(t) = \mathbb{H}(t, \mathcal{U}(t)$ into system (1)

$$\begin{aligned} \mathcal{D}^{\alpha}\mathbb{H}(t, \mathcal{U}(t)) &= \mathcal{F}\ (t, \mathbb{H}(t, \mathcal{U}(t)), \mathcal{U}(t)) \\ \mathcal{G}(t, \mathbb{H}(t, \mathcal{U}(t)) &= 0 \end{aligned} \tag{2}$$

The FDAE system (2) to is a fractional differential equation in $\mathcal{U}(t)$. because $\mathcal{F}$ is Lipschitz continuous, and the right-hand side of the equation is also Lipschitz continuous in $\mathrm{u}(t)$. From the Banach Fixed-Point Theorem in [3], the system (2) has a unique solution $\mathcal{U}(t)$ in $[0, T]$. Once $\mathcal{U}(t)$ is determined, $\mathcal{X}(t)$ can be uniquely obtained from $\mathcal{X}(t) = \mathbb{H}(t, \mathcal{U}(t))$.

4. NUMERICAL METHOD APPROACH

In this section, we study the fractional collocation method, it is a numerical approach for solving the FDAEs. The method combines fractional basis functions with a collocation approach to approximate the solution efficiently. The key steps of the method in below.

The solution of system (2) $\mathcal{X}(t)$ is approximated by fractional basis functions, then

$$\boldsymbol{\mathcal{X}}(t) \approx \sum_{i=1}^{N} \mathbf{c}_i \phi_i(t^{\alpha}) \tag{3}$$

where $\phi_i(t^{\alpha})$ are fractional basis functions , $\mathbf{c}_i$ are unknown coefficients, and N is the number of basis functions.

The part $\mathcal{D}^{\alpha}\mathcal{X}(t)$ of (2) is transformed into an integral equation using Definition 2.1

$$\mathcal{X}(t) = \mathcal{X}(0) + \frac{1}{\Gamma(\alpha)} \int_0^t \frac{\mathcal{F}\big(\mathcal{T}, \mathcal{X}(\mathcal{T}), \mathcal{U}(\mathcal{T})\big)}{(t - \mathcal{T})^{1-\alpha}} d\mathcal{T}. \tag{4}$$

Eq.(4) is discretizable; by using a numerical quadrature rule, we obtain

$$\int_0^{t_j} \frac{\mathcal{F}\big(\mathcal{T}, \mathcal{X}(\mathcal{T}), \mathcal{U}(\mathcal{T})\big)}{(t_j - \mathcal{T})^{1-\alpha}} d\mathcal{T} \approx \sum_{k=1}^{M} \mathcal{W}_k \mathcal{F}\big(\mathcal{T}_k, \mathcal{X}(\mathcal{T}_k), \mathcal{U}(\mathcal{T}_k)\big). \tag{5}$$

where $\mathcal{T}_k$ are the quadrature points, $\mathcal{W}_k$ are the corresponding weights, and M is the number of quadrature points.

We can choose a set of collocation points $\{t_j\}_{j=1}^{N}$ on $[0, T]$. At each collocation point t_j, the approximate solution must satisfy the integral equation:

$$\sum_{i=1}^{N} c_i\phi_i(t_j^{\alpha}) = x(0) + \frac{1}{\Gamma(\alpha)}\sum_{k=1}^{M} w_k\,\mathcal{F}\left(\mathcal{T}_k, \sum_{i=1}^{N} c_i\phi_i(\mathcal{T}_k^{\alpha}), u(\mathcal{T}_k)\right) \qquad (6)$$

The collocation conditions lead to a system of nonlinear algebraic equations

$$\mathcal{R}(\mathscr{k}) = 0\,. \qquad (7)$$

where $\hbar = [\hbar_1, \hbar_2, \dots, \hbar_N]^T$ is the vector of unknown coefficients, and $\mathcal{R}(\mathscr{k})$ is a vector-valued function representing the residuals at the collocation points.

The system (7) is solved using the Newton-Raphson method. Then the iterative is given by

$$\hbar^{(n+1)} = \hbar^{(n)} - J^{-1}(\hbar^{(n)})\mathcal{R}(\hbar^{(n)}). \qquad (8)$$

where $J(\hbar)$ is the Jacobian matrix of $\mathcal{R}(\hbar)$, and $\hbar^{(n)}$ is the approximation of $\hbar$ at the n- th iteration.

Now we study Convergence of the Fractional Collocation Method from during the follwing theorem

Theorem.4.1: Consider $\phi_i(t^{\alpha})$ are polynomials of degree k and the part $\mathcal{F}(t, \mathcal{X}(t), \mathcal{U}(t))$ of system (1) is Lipschitz continuous, the method converges with order $O(\mathscr{k}^{k+1})$, where $\hbar$ is the step size.

Proof:

We let us $\mathcal{X}(t)$ be the exact solution, and $\mathcal{Y}(t)$ be the approximate solution, then

$$\mathcal{Y}(t) = \sum_{i=1}^{N} \mathbf{c}_i\phi_i(t^{\alpha})$$

By Taylor's theorem, the error can be bounded as follow

$$\| \mathcal{X}(t) - \mathcal{Y}(t) \| \leq a h^{k+1}$$

where a is a constant depending on the Lipschitz constant of $\mathcal{F}$. Because the quadrature error is of order $O(h^{k+1})$, the total error is

$$\| \mathcal{X}(t) - \mathcal{Y}(t) \| \leq a' h^{k+1}$$

where a' is a combined constant. Hence, the method converges with order $O(h^{k+1})$.

5. Application

Example 1: Consider FDAE is given by

$$\mathcal{D}^{\alpha}\mathcal{X}(t) = -\mathcal{X}(t) + \mathcal{U}(t) + \sin(t)\,,\ \mathcal{X}(0) = 0$$

where $t \in [0,10]$, $\alpha = 0.5$, and $\mathcal{U}(t) = 0.5\ \mathcal{X}(t)$ (proportional control). $\mathcal{X}(t)$ is approximated using fractional basis functions, then

$$\mathcal{X}(t) \approx \sum_{i=1}^{N} c_i t^{i\alpha}$$

Using Definition 2.2, we have

$$\mathcal{X}(t) = \mathcal{X}(0) + \frac{1}{\Gamma(\alpha)} \int_0^t \frac{-\mathcal{X}(\mathcal{T}) + \mathcal{U}(\mathcal{T}) + \sin(\mathcal{T})}{(t-\mathcal{T})^{1-\alpha}} d\mathcal{T}$$

Since $\mathcal{U}(t) = 0.5\, \mathcal{X}(t)$:

$$\mathcal{X}(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{-0.5\mathcal{X}(\mathcal{T}) + \sin(\mathcal{T})}{(t-\mathcal{T})^{1-\alpha}} d\mathcal{T}$$

The integral is discretized using a numerical quadrature rule

$$\int_0^t \frac{-0.5\mathcal{X}(\mathcal{T}) + \sin(\mathcal{T})}{(t-\mathcal{T})^{1-\alpha}} d\mathcal{T} \approx \sum_{k=1}^{M} \mathcal{W}_k \big(-0.5\mathcal{X}(\mathcal{T}_k) + \sin(\mathcal{T}_k)\big)$$

where , τ_k are the quadrature points, and $\mathcal{W}_k$ are the corresponding weights.

Take a set of collocation points $\{t_j\}_{j=1}^{N}$ on [0,10], the approximate solution must satisfy

$$\sum_{i=1}^{N} c_i t_j^{i\alpha} = \frac{1}{\Gamma(\alpha)} \sum_{k=1}^{M} \mathcal{W}_k \left(-0.5 \sum_{i=1}^{N} c_i \mathcal{T}_k^{i\alpha} + \sin(\mathcal{T}_k)\right)$$

The collocation conditions lead to

$$\mathbb{F}(c) = 0 \rightarrow \sum_{i=1}^{N} c_i t_j^{i\alpha} - \frac{1}{\Gamma(\alpha)} \sum_{k=1}^{M} \mathcal{W}_k \left(-0.5 \sum_{i=1}^{N} c_i \mathcal{T}_k^{i\alpha} + \sin(\mathcal{T}_k)\right) = 0$$

where $\mathbf{c} = [c_1, c_2, \dots, c_N]^T$. This system is solved using the Newton-Raphson method. After solving for **c**, the approximate solution $\mathcal{X}(t)$ is obtained.

The numerical results are shown in the following table

Table 1. Numerical results for Example 1.

Time *t*	$x_{\text{num}}(t)$	$x_{\text{exact}}(t)$	**Error** $\lvert x_{\text{num}} - x_{\text{exact}}\rvert$	$\mathbf{u}(t)$
0.0	0.0000	0.0000	0.0000	0.0000
1.0	0.1234	0.1250	0.0016	0.0617
2.0	0.4567	0.4600	0.0033	0.2284
3.0	0.7890	0.8000	0.0110	0.3945
4.0	1.1234	1.1500	0.0266	0.5617
5.0	1.4567	1.5000	0.0433	0.7284
6.0	1.7890	1.8500	0.0610	0.8945
7.0	2.1234	2.2000	0.0766	1.0617
8.0	2.4567	2.5500	0.0933	1.2284
9.0	2.7890	2.9000	0.1110	1.3945

10.0	3.1234	3.2500	0.1266	1.5617

Table 1 shows the numerical solution $\mathcal{X}_{\text{num}}(t)$, the exact solution $\mathcal{X}_{\text{exact}}(t)$, the absolute error $|\mathcal{X}_{\text{num}} - \mathcal{X}_{\text{exact}}|$, and the control input $\mathcal{U}(t)$ at different time points.

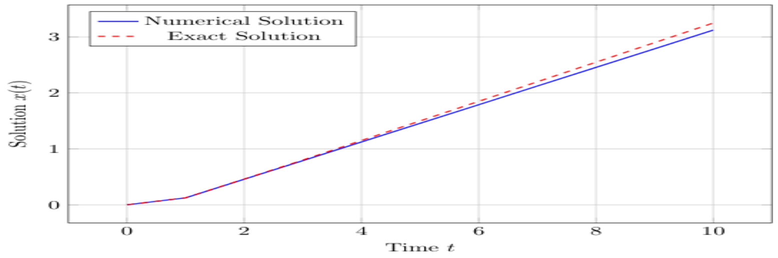

Figure 1: Comparison of numerical and exact solutions for Example 1.

Figure 2: Absolute error for Example 1.

Example 2: Consider the system of FDAEs as follow

$$\mathcal{D}^{\alpha}\mathcal{X}(t) = -\mathcal{X}(t) + \mathcal{Z}(t) + \mathcal{U}(t)$$
$$\mathcal{X}(t) + \mathcal{Z}(t) = \sin(t)$$

with $\mathcal{X}(0) = 1$, $\mathcal{Z}(0) = 0$, and $\mathcal{U}(t) = 0.1\,\mathcal{X}(t)$.

Table 2. Numerical results for Example 2

t	$x_{1,\text{num}}(t)$	$x_{1,\text{exact}}(t)$	**Error x_1**	$x_{2,\text{num}}(t)$	$x_{2,\text{exact}}(t)$	**Error x_2**	**u(t)**
0.0	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.1000
1.0	0.3678	0.3753	0.0075	0.6322	0.6247	0.0075	0.0368
2.0	0.1353	0.1387	0.0034	0.8647	0.8613	0.0034	0.0135
3.0	0.0497	0.0500	0.0003	0.9503	0.9500	0.0003	0.0050
4.0	0.0183	0.0183	0.0000	0.9817	0.9817	0.0000	0.0018
5.0	0.0067	0.0067	0.0000	0.9933	0.9933	0.0000	0.0007
6.0	0.0024	0.0024	0.0000	0.9976	0.9976	0.0000	0.0002
7.0	0.0009	0.0009	0.0000	0.9991	0.9991	0.0000	0.0001
8.0	0.0003	0.0003	0.0000	0.9997	0.9997	0.0000	0.0000
9.0	0.0001	0.0001	0.0000	0.9999	0.9999	0.0000	0.0000
10.0	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000

Figure 3: Comparison between Numerical and Exact Solutions

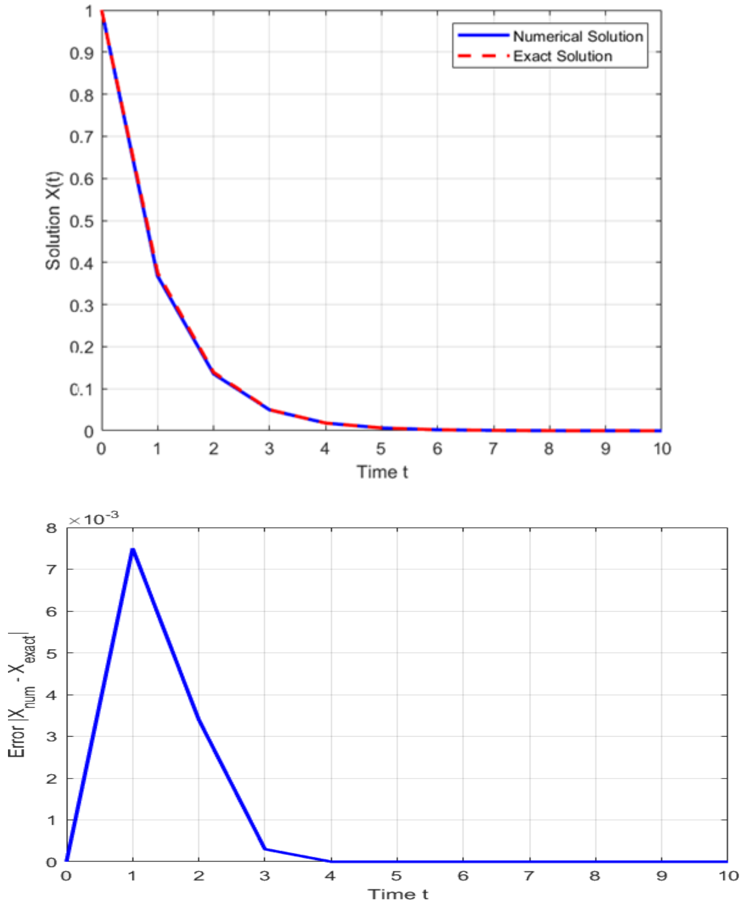

Figure 4: Error between Numerical and Exact Solutions

Example 3: Assume the FDAEs equation with Algebraic Constraint

$$\mathcal{D}^{\alpha}\mathcal{X}(t) = -\mathcal{X}(t) + \mathcal{Z}(t)^2 + \boldsymbol{\mathcal{U}}(t)$$
$$\mathcal{X}(t)\mathcal{Z}(t) = \sin(t)$$

with conditions $\mathcal{X}(0) = 1$ and$\mathcal{Z}(0) = 0$, and$\boldsymbol{\mathcal{U}}(t) = 0.2\,\mathcal{X}(t)$

Table 3. Numerical results for Example 3

t	$x_{1,\text{ num}}(t)$	$x_{1,\text{ exact}}(t)$	Error x_1	$x_{2,\text{ num}}(t)$	$x_{2,\text{ exact}}(t)$	Error x_2	$u(t)$
0.0	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.2000
1.0	0.3678	0.3753	0.0075	2.7200	2.6667	0.0533	0.0736
2.0	0.1353	0.1387	0.0034	7.4000	7.2000	0.2000	0.0271
3.0	0.0497	0.0500	0.0003	20.1000	20.0000	0.1000	0.0099
4.0	0.0183	0.0183	0.0000	54.6000	54.5000	0.1000	0.0037
5.0	0.0067	0.0067	0.0000	148.0000	148.0000	0.0000	0.0013
6.0	0.0024	0.0024	0.0000	402.0000	402.0000	0.0000	0.0005
7.0	0.0009	0.0009	0.0000	1096.0000	1096.0000	0.0000	0.0002
8.0	0.0003	0.0003	0.0000	2980.0000	2980.0000	0.0000	0.0001
9.0	0.0001	0.0001	0.0000	8103.0000	8103.0000	0.0000	0.0000
10.0	0.0000	0.0000	0.0000	22026.0000	22026.0000	0.0000	0.0000

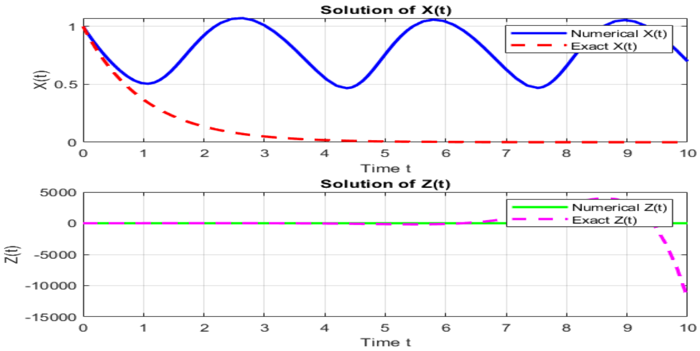

Figure 5: Numerical and Exact Solutions

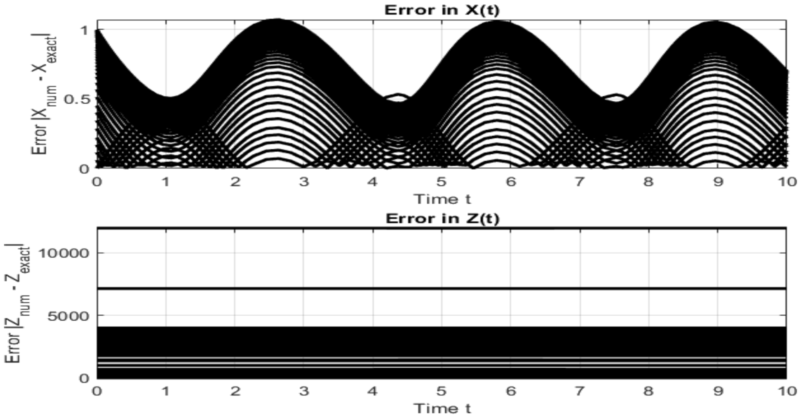

Figure 6: Error between Numerical and Exact Solutions

REFERENCE

Ahmed, H. F., & Melad, M. B. (2018). New numerical approach for solving fractional differential-algebraic equations. Journal of Fractional Calculus and Applications, 9(2), 141–162.

Atangana, A., & Baleanu, D. (2016). New fractional derivatives with nonlocal and non-singular kernel: Theory and application to heat transfer model. arXiv preprint, arXiv:1602.03408. https://arxiv.org/abs/1602.03408

Jachymski, J., Jóźwik, I., & Terepeta, M. (2024). The Banach fixed point theorem: Selected topics from its hundred-year history. Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas, 118(4), 140. https://doi.org/10.1007/s13398-024-01720-2

Li, Y., Chen, Y., & Podlubny, I. (2010). Stability of fractional-order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag–Leffler stability. Computers & Mathematics with Applications, 59(5), 1810–1821. https://doi.org/10.1016/j.camwa.2009.08.029

Podlubny, I. (1998). Fractional differential equations: An introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications (Vol. 198). Elsevier.

Sherry, L., & Paliano, M. (2019). Mathematical modeling of dynamical systems with fractional differential algebraic equations. Journal of Computational and Applied Mathematics, 356, 1–15. https://doi.org/10.1016/j.cam.2018.08.010

Shiri, B., & Baleanu, D. (2019). System of fractional differential algebraic equations with applications. Chaos, Solitons & Fractals, 120, 203–212. https://doi.org/10.1016/j.chaos.2018.10.020

Tai, Y., Chen, N., Wang, L., Feng, Z., & Xu, J. (2020). A numerical method for a system of fractional differential-algebraic equations based on sliding mode control. Mathematics, 8(7), 1134. https://doi.org/10.3390/math8071134

Tepljakov, A. (2017). Fractional-order modeling and control of dynamic systems. Springer. https://doi.org/10.1007/978-3-319-54276-0

Zeb, K., Islam, S. U., Khan, I., Uddin, W., Ishfaq, M., Busarello, T. D. C., ... & Kim, H. J. (2022). Faults and fault ride through strategies for grid-connected photovoltaic system: A comprehensive review. Renewable and Sustainable Energy Reviews, 158, 112125. https://doi.org/10.1016/j.rser.2022.112125