Bilangan : Jurnal Ilmiah Matematika, Kebumian dan Angkasa
Volume. 3 Nomer. 2, April 2025
e-ISSN :3032-7113; p-ISSN :3032-6389, Hal 74-83
DOI: https://doi.org/10.62383/bilangan.v3i2.462
Available online at: https://journal.arimsi.or.id/index.php/Bilangan

Infinity Theory

Ahmed Mezher Marir
Al-Mustansiriyah University, College of Education, Department of Mathematics, Baghdad, Iraq.

Correspondence Author: ahmedmezher1402@gmail.com

Abstract. *This research defines Infinity, Inverse of Infinity, and Inverse of zero, the concept of approach, the symbol $(a, b){:}\, y$ and show that, Infinity theory: i- Infinity is a natural number i.e ($\infty \in N$). ii- The Inverse of infinity is not equal to zero and the Inverse of zero is not equal to infinity ($\frac{1}{\infty} \neq 0, \frac{1}{0} \neq \infty$). Also, show that if $\frac{1}{\infty}$ is real number, then $\frac{1}{\infty}$ equals zero. And the set of real numbers is equals to the set of rational numbers ($R = Q$).*

Keywords: *Infinity, Inverse of Infinity, Inverse of zero, the symbol $(a, b){:}\, y$ and the concept of approach.*

1. INTRODUCTION

Despite the common misconception that infinity and zero are opposites, they actually have a connection. The sign 0 represents emptiness, whereas ∞ represents intangible quantities. Finitude is a concept that falls between pure nothingness (0) and infiniteness. Some philosophers who debate the existence of an infinite set may not be conversant with the mathematical literature that defines these notions explicitly and rigorously (Maria et al. 2010) . (Tall 1996) I think mathematicians should be involved since analysts have defined and utilised infinity in a manner that is pertinent to philosophers' arguments. (Vollrath 1987)

The basic definition of infinity is the sum of all natural numbers and the closed period.[0,1] is Infinity, and we can also define Infinity in the following way: If $M \geq n, \forall\, n \in N$, then $M = \infty$. Inverse of infinity is not real number, also Inverse of infinity is not zero. The result of dividing one by infinity is equal to zero and the remainder is one (since the divisor is infinity) and this concept differs from zero, i.e., if the result of division is zero and the remainder is zero (and the divisor is infinity or something else). It can be said that the result of dividing one by infinity or the Inverse of infinity is approaching zero from the right side ($\frac{\mathbf{1}}{\infty} \to 0_+$) i.e $\frac{1}{\infty}$ approaches 0 from the right.

Infinity theory 1.1:

i- Infinity is a natural number i.e ($\infty \in N$).

ii- The inverse of infinity and 0 are not comparable ($\frac{1}{\infty} \neq 0$) and $\left(\frac{1}{0} \neq \infty\right)$

Received: Maret 17, 2025; Revise:d Maret 31,2025; Accepted: April 07,2025; Published: April 09, 2025

Proof:

Let $N = \{1, 2, 3, \dots\}$ is Natural numbers

$(\forall\, n \in N, \exists\, n + 1 \in N\,)$Such that

Let $b_i = \frac{1}{2^i}\,, \forall\, i \in N$

$B = \{b_i, i \in N\} = \{b_1, b_2, b_3, \dots\}$

We impose

$b = The\ sum\ of\ the\ elements\ of\ set\ B$

$= b_1 + b_2 + b_3 + \cdots$

And we can conclude $b = 1\ \ or\ \ b \to 1_-$ i.e b approaches 1 from the left and this more correct.

Let $A = \{a_i, i \in N\} = \{a_1, a_2, a_3, \dots\}$

$a = The\ sum\ of\ the\ elements\ of\ set\ A$

$= a_1 + a_2 + a_3 + \cdots = 1$

We impose

$a_i = a_j = a_0, \forall\, i, j\ \in N, a = 1$

$\Longrightarrow\ \ 1 = a_0 + a_0 + a_0 + \cdots$

And we can conclude $a_i = a_0 = \frac{1}{\infty}\,, \forall\, i \in N$

Let $c_i = b_i - a_i\,, \forall\, i \in N$

$C = \{c_i, i \in N\} = \{c_1, c_2, c_3, \dots\}$

We impose

$c = The\ sum\ of\ the\ elements\ of\ set\ C$

$= c_1 + c_2 + c_3 + \cdots$

$c_i = b_i - a_i\,, \forall\, i \in N$

$\Longrightarrow\ c = (b_1 - a_1) + (b_2 - a_2) + (b_3 - a_3) + \cdots$

$= (b_1 + b_2 + b_3 + \cdots) - (a_1 + a_2 + a_3 + \cdots)$

And we can conclude

$c = b - a\ \Longrightarrow\ c = 0\ \ or\ \ c \to 0_-$

(i.e c approaches 0 from the left)

Now

i- Let $\infty \notin N$, i.e if $i \in N, then\ \ i\ \neq \infty$

$$\Longrightarrow\ c_i = b_i - a_i = \frac{1}{2^i} - \frac{1}{\infty} > 0\,, \forall\, i \in N$$

$\Longrightarrow\ \ c_1 + c_2 + c_3 + \cdots = c > 0\ \ \blacksquare$

And this is a contradiction

$\Longrightarrow\ \infty \in N$

or

If $c_i > 0, \forall\ i \in N$

$\Longrightarrow\ c > 0\ \ but\ \ c = 0\ \ or\ \ c \to 0_-$

This means that there is $c_j \leq 0$ such that $j \in N$

$$\Longrightarrow\ \ b_j - a_j \leq 0$$

$\Longrightarrow\ \dfrac{1}{2^j} - \dfrac{1}{\infty} \leq 0\ \ \Longrightarrow\ \dfrac{1}{2^j} \leq \dfrac{1}{\infty}\ \ \Longrightarrow\ \ 2^j \geq \infty\ \Longrightarrow j = \infty$

$\Longrightarrow\ \infty \in N$

ii- Let $\frac{1}{\infty} = 0$

$\Longrightarrow\ \ a_0 = 0\ \ \Longrightarrow\ \ c_i = b_i - a_i = b_i - a_0 = b_i$

$\Longrightarrow\ \ c = b\ \Longrightarrow\ b = b - a\ \Longrightarrow a = 0\ \ \blacksquare$

And this is a contradiction

since $a = 1$ So we conclude that $\frac{1}{\infty} \neq 0$

Let $\frac{1}{0} = \infty\ \ \Longrightarrow\ \ \dfrac{\mathbf{1}}{\left(\frac{\mathbf{1}}{\mathbf{0}}\right)} = \dfrac{\mathbf{1}}{\infty}\ \ \Longrightarrow\ \ 0 = \dfrac{\mathbf{1}}{\infty}$

But $\frac{1}{\infty} \neq 0\ \ \Longrightarrow\ \ \frac{1}{0} \neq \infty$

Then $\frac{1}{\infty} \neq 0\ \ \left(or\ \ \frac{1}{\infty} \to 0_+\right)$

Remark 1.2*:*

Let $a_1 + a_2 + a_3 + \cdots + a_n = 1$

$n \to \infty$

$a_1 = a_2 = a_3 = \cdots = a_n = a_0$

$\Longrightarrow\ a_0 = \dfrac{1}{n}$

$b = b_1 + b_2 + b_3 + \cdots$

Such that $b_i = \frac{1}{2^i}$, $\forall\ i \in N$

$$b = 1 - \frac{1}{2^n}$$

$$c = b - a = \frac{-1}{2^n}$$

$$c = \lim_{n\to\infty} \frac{-1}{2^n} = 0$$

i.e $c = 0 \ or \ c \to 0_-$

Theorem 1.3:

Real and rational numbers are equivalent

$(R = Q)$.

Proof:

$Q \subseteq R$ To prove $R \subseteq Q$, let $m \in R$, then

$$m = \mp\left(n + \frac{n_1}{10} + \frac{n_2}{10^2} + \frac{n_3}{10^3} + \cdots + \frac{n_r}{10^r} + \cdots + \frac{n_\infty}{10^\infty},\right)$$

Such that $r \to \infty$ (r *is approaching* ∞) *or* $r = \infty$

$n \in W = \{0,1,2,\dots\}$,

$n_1, n_2, n_3, \dots, n_r, \dots, n_\infty \in \{0,1,2,\dots,9\}$

$$\Longrightarrow \ n + \frac{n_1}{10} + \frac{n_2}{10^2} + \frac{n_3}{10^3} + \cdots + \frac{n_r}{10^r} + \cdots + \frac{n_\infty}{10^\infty},$$

$$= \frac{n \times 10^r}{10^r} + \frac{n_1 \times 10^{r-1}}{10^r} + \frac{n_2 \times 10^{r-2}}{10^r} + \frac{n_3 \times 10^{r-3}}{10^r}$$

$$+ \cdots + \frac{n_r \times 10^0}{10^r}$$

$$= \frac{n \times 10^r + n_1 \times 10^{r-1} + n_2 \times 10^{r-2} + n_3 \times 10^{r-3} + \cdots + n_r}{10^r}$$

$0 \leq n \times 10^r + n_1 \times 10^{r-1} + n_2 \times 10^{r-2}$

$+ n_3 \times 10^{r-3} + \cdots + n_r \leq \infty$

$\Longrightarrow \ \mp(n \times 10^r + n_1 \times 10^{r-1} + n_2 \times 10^{r-2}$

$+ n_3 \times 10^{r-3} + \cdots + n_r) \in Z$

And

$10^r \to \infty \ or \ 10^r = \infty \ \Longrightarrow \ 10^r \in Z$

$\Longrightarrow \ m \in Q \ \Longrightarrow \ R \subseteq Q \ \Longrightarrow \ R = Q$

Remark 1.4:

$$\frac{x}{y} = (a, b) \colon y$$

In other words, the result of dividing x by y is equal to a and the remainder is b, noting that the divisor is y.

$x = ay + b$

$$(a, b)\text{: y} = \frac{ay + b}{y} = a + \frac{b}{y}$$

For example:

$$\frac{10}{3} = (3,1){:}\,3\,, \frac{1}{3} = (0,1){:}\,3\,, \frac{9}{3} = (3,0){:}\,3 = 3$$

Remark 1.5:

$(a, 0)\text{: y} = a\,, \forall\, a, y \in R^{+}$

$(0,1){:}\, \infty \to 0_{+}$

Approaches from the right

$(a, 1){:}\, \infty \to a_{+}\,, \forall\, a \in R^{+}$

Approaches from the right

$(a, b){:}\, \infty \to a_{+}\,\,, \forall\, a, b \in R^{+}$

Approaches from the right

Lemma 1.6:

i) $0 \times \infty = 0$

ii) If a is positive real number, then $a \times \infty = \infty$

iii) If b is negative real number, then $b \times \infty = -\infty$

Remark 1.7:

$$\frac{a}{b} = c \;\Leftrightarrow\; a = b \times c\,, \qquad \forall\, a, b, c$$

Theorem 1.8:

If $\frac{1}{\infty}$ is real number, then $\frac{1}{\infty}$ equals zero.

Proof:

Let $\frac{1}{\infty}$ is real number

$\Longrightarrow$ $\frac{1}{\infty}$ is positive or negative or zero

Let $\frac{1}{\infty} = a$, a is positive, by remark 1.7 and Lemma 1.6, then

$1 = \infty \times a \;\Longrightarrow\; 1 = \infty$

Contradiction

Let $\frac{1}{\infty} = b$, b is negative, by remark 1.7 and Lemma 1.6, then

$1 = \infty \times b \implies 1 = -\infty$

Contradiction

we can conclude $\frac{1}{\infty} = 0$

Remark 1.9: $\frac{1}{\infty}$ is not real number, $\frac{1}{\infty}$ is not complex number, $\frac{1}{\infty}$ is different concept and $\frac{1}{\infty} \to 0_+$

Remark 1.10:

Let $H = \{h : h \text{ is Infinity}\}$

$$\frac{1}{0} > h, \forall\, h \in H$$

Remark 1.11:

there are $\infty_1, \infty_2, \infty_3, \ldots \in H$

Such that $\infty_1 + \infty_2 = \infty_3, \infty_4 - \infty_5 = \infty_6$

$\infty_7 \times \infty_8 = \infty_9, \infty_{10} \div \infty_{11} = \infty_{12}, \infty_{13} = \infty_{14} + 5$

$\infty_{15} = 7\infty_{16}, \quad \infty_{17} > \infty_{18}, \infty_{19} = 2^{\infty_{20}} \ldots$

Lemma 1.12:

$$y \to a_+ \iff y = a + \frac{1}{\infty}$$

$$y \to a_- \iff y = a - \frac{1}{\infty}$$

$$y \to a \iff y = a \mp \frac{1}{\infty}$$

Let $s = \mp\frac{1}{\infty}, s_- = \frac{-1}{\infty}$, $s_+ = \frac{1}{\infty}$, then

$y \to a \iff y = a + s$

$y \to a_- \iff y = a + s_-$

$y \to a_+ \iff y = a + s_+$

2.1 Limits: (Sierpińska 1987)

We say that L is a right hand limit for $f(x)$ when x approaches c for the right, written $\lim_{x \to c_+} f(x) = L$.

Similary, L is the left hand limit for $f(x)$ when x approaches c for the left, written $\lim_{x\to c_-} f(x) = L$, then $\lim_{x\to c} f(x) = L$ if and only if $\lim_{x\to c_+} f(x) = \lim_{x\to c_-} f(x)$

Basic Rational Function Limit Theorem: (Szydlik 2000; Tall and Vinner 1981)

If f is rational and a∈Dom(f), then lim┬(x→a) f(x)=f(a). For limit evaluation, use (plug in) $x = a$, and evaluate $f(a)$.

A Two-Sided Limit at a Point:

$$\lim_{x\to a} f(x) = L \iff \left[\lim_{x\to a^-} f(x) = L \text{ and } \lim_{x\to a^+} f(x) = L\right], (a, L \in R)$$

A two-sided limit exists, with equal left and right-hand limitations.

If one or both one-sided limits are unequal, the two-sided limit does not exist.

Theorem of Extended Limit for Rational Functions:

If f is rational and a∈Dom(f), then

$\lim_{x\to a^-} f(x) = f(a)$ and $\lim_{x\to a^+} f(x) = f(a) \implies \lim_{x\to a} f(x) = f(a)$

Enter x=a and evaluate f(a) to assess each limit.

The idea of inverse infinity (or infinity) can be used to solve some limits, as follows:

Example 2.2: solve the following limit.

$$\lim_{x\to 0} \frac{1}{x^2+1}$$

Sol:

By lemma 1.12

$$x \to 0_+ \implies x = \frac{1}{\infty}$$

By remark 1.11

$$\frac{1}{x^2+1} = \frac{1}{\left(\frac{1}{\infty}\right)^2 + 1} = \frac{1}{\frac{1}{\infty^2}+1} = \frac{1}{\frac{1}{\infty_1}+1}$$

$$\infty_1 = \infty^2$$

$$= \frac{1}{\left(\frac{1+\infty_1}{\infty_1}\right)} = \frac{\infty_1}{1+\infty_1} = \frac{\infty_1 + 1 - 1}{1+\infty_1}$$

$$= 1 - \frac{1}{\infty_1 + 1} = 1 - \frac{1}{\infty_2}$$

$\infty_2 = \infty_1 + 1$

$$\Longrightarrow \ \frac{1}{x^2+1} \to 1_- \ \Longrightarrow \ \lim_{x\to 0}\frac{1}{x^2+1} = 1$$

Remark 2.3:

If $f(x) = b \ \ or \ \ f(x) \to b \ \ \ such \ that \ or \ when \ \ x \to a$

$\lim_{x\to a} f(x) = b$, then

Example 2.4: find the following function limit.

$$\lim_{x\to 1}\frac{x^2-1}{x-1}$$

Sol:

By lemma 1.12

$$x \to 1 \quad \Longrightarrow \quad x = 1 \mp \frac{1}{\infty}$$

$$\frac{x^2-1}{x-1} = \frac{\left(1 \mp \frac{1}{\infty}\right)^2 - 1}{1 \mp \frac{1}{\infty} - 1} = \frac{1 \mp \frac{2}{\infty} + \frac{1}{\infty^2} - 1}{1 \mp \frac{1}{\infty} - 1}$$

$$= \frac{\left(\mp \frac{2}{\infty} + \frac{1}{\infty^2}\right)}{\left(\mp \frac{1}{\infty}\right)} = \frac{\left(\mp \frac{1}{\infty}\right)\left(2 \mp \frac{1}{\infty}\right)}{\left(\mp \frac{1}{\infty}\right)} = 2 \mp \frac{1}{\infty}$$

$$\Longrightarrow \ \frac{x^2-1}{x-1} \to 2 \ \Longrightarrow \ \lim_{x\to 1}\frac{x^2-1}{x-1} = 2$$

Example 2.5: solve the following limit.

$$\lim_{x\to 0}\frac{1}{x^2-1}$$

Sol:

By lemma 1.12

$$x \to 0 \quad \Longrightarrow \quad x = \mp \frac{1}{\infty}$$

$$\frac{1}{x^2-1} = \frac{1}{\left(\mp \frac{1}{\infty}\right)^2 - 1} = \frac{1}{\frac{1}{\infty^2} - 1} = \frac{1}{\frac{1}{\infty_1} - 1}$$

By remark 1.12

$\infty_1 = \infty^2$

$$= \frac{1}{\left(\frac{1-\infty_1}{\infty_1}\right)} = \frac{\infty_1}{1-\infty_1} = -\frac{\infty_1}{\infty_1 - 1}$$

$$= -\frac{\infty_1 - 1 + 1}{\infty_1 - 1} = -\left(1 + \frac{1}{\infty_1 - 1}\right)$$

$$= -\left(1 + \frac{1}{\infty_1 - 1}\right) = -\left(1 + \frac{1}{\infty_2}\right) = -1 - \frac{1}{\infty_2}$$

$$\infty_2 = \infty_1 - 1$$

$$\Rightarrow \ \frac{1}{x^2-1} \to -1 \quad \Rightarrow \quad \lim_{x\to 0} \frac{1}{x^2-1} = -1$$

Example 2.6: find the following function limit.

$$\lim_{x\to 1} \frac{1}{x^2-1}$$

Sol:

By lemma 1.12

$$x \to 1 \quad \Rightarrow \quad x = 1 \mp \frac{1}{\infty}$$

$$\frac{1}{x^2-1} = \frac{1}{\left(1 \mp \frac{1}{\infty}\right)^2 - 1} = \frac{1}{1 \mp \frac{2}{\infty} + \frac{1}{\infty^2} - 1}$$

$$= \frac{1}{\mp \frac{2}{\infty} + \frac{1}{\infty^2}} = \frac{1}{\frac{1}{\infty}\left(\mp 2 + \frac{1}{\infty}\right)} = \frac{\infty}{\mp 2 + \frac{1}{\infty}} = \mp\infty$$

The limit does not exist

Because the limit of the right is not equal to the limit of the left

The limit of the right

$$\mathrm{x} \to a_+ \ \Leftrightarrow \ x = a + \frac{1}{\infty}$$

The limit of the left

$$\mathrm{x} \to a_- \ \Leftrightarrow \ x = a - \frac{1}{\infty}$$

Example 2.7: solve the following limit.

$$\lim_{x\to 0} \frac{|x|}{x}$$

Sol:

The limit of the right, By lemma 1.12

$$x \to 0_+ \iff x = \frac{1}{\infty}$$

$$\frac{|x|}{x} = \frac{\left|\frac{1}{\infty}\right|}{\frac{1}{\infty}} = \frac{\left(\frac{1}{\infty}\right)}{\left(\frac{1}{\infty}\right)} = 1$$

The limit of the left, By lemma 1.12

$$x \to 0_- \iff x = \frac{-1}{\infty}$$

$$\frac{|x|}{x} = \frac{\left|\frac{-1}{\infty}\right|}{\frac{-1}{\infty}} = \frac{\left(\frac{1}{\infty}\right)}{\left(\frac{-1}{\infty}\right)} = -1$$

Right and left have different limits.

The limit is nonexistent.

REFERENCES

Maria, Kattou, Michael Thanasia, Kontoyianni Katerina, Christou Constantinos, and Philippou George. 2010. "TEACHERS'PERCEPTIONS ABOUT INFINITY: A PROCESS OR AN OBJECT?" *CERME 6–WORKING GROUP 10* 1771.

Sierpińska, Anna. 1987. "Humanities Students and Epistemological Obstacles Related to Limits." *Educational Studies in Mathematics* 18(4):371–97.

Szydlik, Jennifer Earles. 2000. "Mathematical Beliefs and Conceptual Understanding of the Limit of a Function." *Journal for Research in Mathematics Education* 31(3):258–76.

Tall, D. 1996. "Thinking about Infinity." *International Handbook of Mathematics Education* 504–11.

Tall, David, and Shlomo Vinner. 1981. "Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity." *Educational Studies in Mathematics* 12(2):151–69.

Vollrath, Hans-Joachim. 1987. "Begriffsbildung Als Schöpferisches Tun Im Mathematikunterricht." *Zentralblatt Für Didaktik Der Mathematik* 19(3):123–27.