



Infinity Theory

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Abstract. This research defines Infinity, Inverse of Infinity, and Inverse of zero, the concept of approach, the symbol $(a, b): y$ and show that, Infinity theory: i- Infinity is a natural number i.e $(\infty \in N)$. ii- The Inverse of infinity is not equal to zero and the Inverse of zero is not equal to infinity $(\frac{1}{\infty} \neq 0, \frac{1}{0} \neq \infty)$. Also, show that if $\frac{1}{\infty}$ is real number, then $\frac{1}{\infty}$ equals zero. And the set of real numbers is equals to the set of rational numbers $(R = Q)$.

Keywords: Infinity, Inverse of Infinity, Inverse of zero, the symbol $(a, b): y$ and the concept of approach.

1. INTRODUCTION

Despite the common misconception that infinity and zero are opposites, they actually have a connection. The sign 0 represents emptiness, whereas ∞ represents intangible quantities. Finitude is a concept that falls between pure nothingness (0) and infiniteness. Some philosophers who debate the existence of an infinite set may not be conversant with the mathematical literature that defines these notions explicitly and rigorously (Maria et al. 2010). (Tall 1996) I think mathematicians should be involved since analysts have defined and utilised infinity in a manner that is pertinent to philosophers' arguments. (Vollrath 1987)

The basic definition of infinity is the sum of all natural numbers and the closed period. $[0,1]$ is Infinity, and we can also define Infinity in the following way: If $M \geq n, \forall n \in N$, then $M = \infty$. Inverse of infinity is not real number, also Inverse of infinity is not zero. The result of dividing one by infinity is equal to zero and the remainder is one (since the divisor is infinity) and this concept differs from zero, i.e., if the result of division is zero and the remainder is zero (and the divisor is infinity or something else). It can be said that the result of dividing one by infinity or the Inverse of infinity is approaching zero from the right side $(\frac{1}{\infty} \rightarrow 0_+)$ i.e $\frac{1}{\infty}$ approaches 0 from the right.

Infinity theory 1.1:

i- Infinity is a natural number i.e $(\infty \in N)$.

ii- The inverse of infinity and 0 are not comparable $(\frac{1}{\infty} \neq 0)$ and $(\frac{1}{0} \neq \infty)$

Proof:

Let $N = \{1, 2, 3, \dots\}$ is Natural numbers

($\forall n \in N, \exists n + 1 \in N$) Such that

Let $b_i = \frac{1}{2^i}, \forall i \in N$

$$B = \{b_i, i \in N\} = \{b_1, b_2, b_3, \dots\}$$

We impose

$$\begin{aligned} b &= \text{The sum of the elements of set } B \\ &= b_1 + b_2 + b_3 + \dots \end{aligned}$$

And we can conclude $b = 1$ or $b \rightarrow 1_-$ i.e b approaches 1 from the left and this more correct.

$$\text{Let } A = \{a_i, i \in N\} = \{a_1, a_2, a_3, \dots\}$$

$$\begin{aligned} a &= \text{The sum of the elements of set } A \\ &= a_1 + a_2 + a_3 + \dots = 1 \end{aligned}$$

We impose

$$a_i = a_j = a_0, \forall i, j \in N, a = 1$$

$$\Rightarrow 1 = a_0 + a_0 + a_0 + \dots$$

And we can conclude $a_i = a_0 = \frac{1}{\infty}, \forall i \in N$

$$\text{Let } c_i = b_i - a_i, \forall i \in N$$

$$C = \{c_i, i \in N\} = \{c_1, c_2, c_3, \dots\}$$

We impose

$$\begin{aligned} c &= \text{The sum of the elements of set } C \\ &= c_1 + c_2 + c_3 + \dots \end{aligned}$$

$$c_i = b_i - a_i, \forall i \in N$$

$$\Rightarrow c = (b_1 - a_1) + (b_2 - a_2) + (b_3 - a_3) + \dots$$

$$= (b_1 + b_2 + b_3 + \dots) - (a_1 + a_2 + a_3 + \dots)$$

And we can conclude

$$c = b - a \Rightarrow c = 0 \text{ or } c \rightarrow 0_-$$

(i.e c approaches 0 from the left)

Now

i- Let $\infty \notin N$, i.e if $i \in N$, then $i \neq \infty$

$$\Rightarrow c_i = b_i - a_i = \frac{1}{2^i} - \frac{1}{\infty} > 0, \forall i \in N$$

$$\Rightarrow c_1 + c_2 + c_3 + \dots = c > 0 \blacksquare$$

And this is a contradiction

$$\Rightarrow \infty \in N$$

or

If $c_i > 0, \forall i \in N$

$$\Rightarrow c > 0 \text{ but } c = 0 \text{ or } c \rightarrow 0_-$$

This means that there is $c_j \leq 0$ such that $j \in N$

$$\Rightarrow b_j - a_j \leq 0$$

$$\Rightarrow \frac{1}{2^j} - \frac{1}{\infty} \leq 0 \Rightarrow \frac{1}{2^j} \leq \frac{1}{\infty} \Rightarrow 2^j \geq \infty \Rightarrow j = \infty$$

$$\Rightarrow \infty \in N$$

ii- Let $\frac{1}{\infty} = 0$

$$\Rightarrow a_0 = 0 \Rightarrow c_i = b_i - a_i = b_i - a_0 = b_i$$

$$\Rightarrow c = b \Rightarrow b = b - a \Rightarrow a = 0 \blacksquare$$

And this is a contradiction

since $a = 1$ So we conclude that $\frac{1}{\infty} \neq 0$

$$\text{Let } \frac{1}{0} = \infty \Rightarrow \frac{1}{\left(\frac{1}{0}\right)} = \frac{1}{\infty} \Rightarrow 0 = \frac{1}{\infty}$$

$$\text{But } \frac{1}{\infty} \neq 0 \Rightarrow \frac{1}{0} \neq \infty$$

Then $\frac{1}{\infty} \neq 0$ (or $\frac{1}{\infty} \rightarrow 0_+$)

Remark 1.2:

Let $a_1 + a_2 + a_3 + \dots + a_n = 1$

$n \rightarrow \infty$

$$a_1 = a_2 = a_3 = \dots = a_n = a_0$$

$$\Rightarrow a_0 = \frac{1}{n}$$

$$b = b_1 + b_2 + b_3 + \dots$$

Such that $b_i = \frac{1}{2^i}, \forall i \in N$

$$b = 1 - \frac{1}{2^n}$$

$$c = b - a = \frac{-1}{2^n}$$

$$c = \lim_{n \rightarrow \infty} \frac{-1}{2^n} = 0$$

i.e $c = 0$ or $c \rightarrow 0_-$

Theorem 1.3:

Real and rational numbers are equivalent

$(R = Q)$.

Proof:

$Q \subseteq R$ To prove $R \subseteq Q$, let $m \in R$, then

$$m = \mp \left(n + \frac{n_1}{10} + \frac{n_2}{10^2} + \frac{n_3}{10^3} + \dots + \frac{n_r}{10^r} + \dots + \frac{n_\infty}{10^\infty}, \right)$$

Such that $r \rightarrow \infty$ (r is approaching ∞) or $r = \infty$

$n \in W = \{0,1,2, \dots\}$,

$n_1, n_2, n_3, \dots, n_r, \dots, n_\infty \in \{0,1,2, \dots, 9\}$

$$\begin{aligned} \Rightarrow n + \frac{n_1}{10} + \frac{n_2}{10^2} + \frac{n_3}{10^3} + \dots + \frac{n_r}{10^r} + \dots + \frac{n_\infty}{10^\infty}, \\ = \frac{n \times 10^r}{10^r} + \frac{n_1 \times 10^{r-1}}{10^r} + \frac{n_2 \times 10^{r-2}}{10^r} + \frac{n_3 \times 10^{r-3}}{10^r} \\ + \dots + \frac{n_r \times 10^0}{10^r} \\ = \frac{n \times 10^r + n_1 \times 10^{r-1} + n_2 \times 10^{r-2} + n_3 \times 10^{r-3} + \dots + n_r}{10^r} \end{aligned}$$

$$0 \leq n \times 10^r + n_1 \times 10^{r-1} + n_2 \times 10^{r-2}$$

$$+ n_3 \times 10^{r-3} + \dots + n_r \leq \infty$$

$$\Rightarrow \mp (n \times 10^r + n_1 \times 10^{r-1} + n_2 \times 10^{r-2}$$

$$+ n_3 \times 10^{r-3} + \dots + n_r) \in Z$$

And

$$10^r \rightarrow \infty \text{ or } 10^r = \infty \Rightarrow 10^r \in Z$$

$$\Rightarrow m \in Q \Rightarrow R \subseteq Q \Rightarrow R = Q$$

Remark 1.4:

$$\frac{x}{y} = (a, b): y$$

In other words, the result of dividing x by y is equal to a and the remainder is b , noting that the divisor is y .

$$x = ay + b$$

$$(a, b): y = \frac{ay + b}{y} = a + \frac{b}{y}$$

For example:

$$\frac{10}{3} = (3,1):3, \frac{1}{3} = (0,1):3, \frac{9}{3} = (3,0):3 = 3$$

Remark 1.5:

$$(a, 0): y = a, \forall a, y \in R^+$$

$$(0,1): \infty \rightarrow 0_+$$

Approaches from the right

$$(a, 1): \infty \rightarrow a_+, \forall a \in R^+$$

Approaches from the right

$$(a, b): \infty \rightarrow a_+, \forall a, b \in R^+$$

Approaches from the right

Lemma 1.6:

i) $0 \times \infty = 0$

ii) If a is positive real number, then $a \times \infty = \infty$

iii) If b is negative real number, then $b \times \infty = -\infty$

Remark 1.7:

$$\frac{a}{b} = c \Leftrightarrow a = b \times c, \quad \forall a, b, c$$

Theorem 1.8:

If $\frac{1}{\infty}$ is real number, then $\frac{1}{\infty}$ equals zero.

Proof:

Let $\frac{1}{\infty}$ is real number

$\Rightarrow \frac{1}{\infty}$ is positive or negative or zero

Let $\frac{1}{\infty} = a$, a is positive, by remark 1.7 and Lemma 1.6, then

$$1 = \infty \times a \Rightarrow 1 = \infty$$

Contradiction

Let $\frac{1}{\infty} = b$, b is negative, by remark 1.7 and Lemma 1.6, then

$$1 = \infty \times b \Rightarrow 1 = -\infty$$

Contradiction

we can conclude $\frac{1}{\infty} = 0$

Remark 1.9: $\frac{1}{\infty}$ is not real number, $\frac{1}{\infty}$ is not complex number, $\frac{1}{\infty}$ is different concept and

$$\frac{1}{\infty} \rightarrow 0_+$$

Remark 1.10:

Let $H = \{h: h \text{ is Infinity}\}$

$$\frac{1}{0} > h, \forall h \in H$$

Remark 1.11:

there are $\infty_1, \infty_2, \infty_3, \dots \in H$

Such that $\infty_1 + \infty_2 = \infty_3, \infty_4 - \infty_5 = \infty_6$

$\infty_7 \times \infty_8 = \infty_9, \infty_{10} \div \infty_{11} = \infty_{12}, \infty_{13} = \infty_{14} + 5$

$\infty_{15} = 7\infty_{16}, \quad \infty_{17} > \infty_{18}, \infty_{19} = 2^{\infty_{20}} \dots$

Lemma 1.12:

$$y \rightarrow a_+ \Leftrightarrow y = a + \frac{1}{\infty}$$

$$y \rightarrow a_- \Leftrightarrow y = a - \frac{1}{\infty}$$

$$y \rightarrow a \Leftrightarrow y = a \mp \frac{1}{\infty}$$

Let $s = \mp \frac{1}{\infty}, s_- = \frac{-1}{\infty}, s_+ = \frac{1}{\infty}$, then

$$y \rightarrow a \Leftrightarrow y = a + s$$

$$y \rightarrow a_- \Leftrightarrow y = a + s_-$$

$$y \rightarrow a_+ \Leftrightarrow y = a + s_+$$

2.1 Limits: (Sierpińska 1987)

We say that L is a right hand limit for $f(x)$ when x approaches c

for the right, written $\lim_{x \rightarrow c_+} f(x) = L$.

Similarly, L is the left hand limit for $f(x)$ when x approaches c for the left, written

$$\lim_{x \rightarrow c^-} f(x) = L, \text{ then } \lim_{x \rightarrow c} f(x) = L \text{ if and only if } \lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^-} f(x)$$

Basic Rational Function Limit Theorem: (Szydlik 2000; Tall and Vinner 1981)

If f is rational and $a \in \text{Dom}(f)$, then $\lim_{x \rightarrow a} f(x) = f(a)$. For limit evaluation, use (plug in) $x = a$, and evaluate $f(a)$.

A Two-Sided Limit at a Point:

$$\lim_{x \rightarrow a} f(x) = L \Leftrightarrow \left[\lim_{x \rightarrow a^-} f(x) = L \text{ and } \lim_{x \rightarrow a^+} f(x) = L \right], (a, L \in \mathbb{R})$$

A two-sided limit exists, with equal left and right-hand limitations.

If one or both one-sided limits are unequal, the two-sided limit does not exist.

Theorem of Extended Limit for Rational Functions:

If f is rational and $a \in \text{Dom}(f)$, then

$$\lim_{x \rightarrow a^-} f(x) = f(a) \text{ and } \lim_{x \rightarrow a^+} f(x) = f(a) \Rightarrow \lim_{x \rightarrow a} f(x) = f(a)$$

Enter $x=a$ and evaluate $f(a)$ to assess each limit.

The idea of inverse infinity (or infinity) can be used to solve some limits, as follows:

Example 2.2: solve the following limit.

$$\lim_{x \rightarrow 0} \frac{1}{x^2 + 1}$$

Sol:

By lemma 1.12

$$x \rightarrow 0_+ \Rightarrow x = \frac{1}{\infty}$$

By remark 1.11

$$\frac{1}{x^2 + 1} = \frac{1}{\left(\frac{1}{\infty}\right)^2 + 1} = \frac{1}{\frac{1}{\infty^2} + 1} = \frac{1}{\frac{1}{\infty_1} + 1}$$

$$\infty_1 = \infty^2$$

$$= \frac{1}{\left(\frac{1 + \infty_1}{\infty_1}\right)} = \frac{\infty_1}{1 + \infty_1} = \frac{\infty_1 + 1 - 1}{1 + \infty_1}$$

$$= 1 - \frac{1}{\infty_1 + 1} = 1 - \frac{1}{\infty_2}$$

$$\infty_2 = \infty_1 + 1$$

$$\Rightarrow \frac{1}{x^2 + 1} \rightarrow 1_- \Rightarrow \lim_{x \rightarrow 0} \frac{1}{x^2 + 1} = 1$$

Remark 2.3:

If $f(x) = b$ or $f(x) \rightarrow b$ such that or when $x \rightarrow a$

$$\lim_{x \rightarrow a} f(x) = b, \text{ then}$$

Example 2.4: find the following function limit.

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

Sol:

By lemma 1.12

$$x \rightarrow 1 \Rightarrow x = 1 \mp \frac{1}{\infty}$$

$$\frac{x^2 - 1}{x - 1} = \frac{\left(1 \mp \frac{1}{\infty}\right)^2 - 1}{1 \mp \frac{1}{\infty} - 1} = \frac{1 \mp \frac{2}{\infty} + \frac{1}{\infty^2} - 1}{1 \mp \frac{1}{\infty} - 1}$$

$$= \frac{\left(\mp \frac{2}{\infty} + \frac{1}{\infty^2}\right)}{\left(\mp \frac{1}{\infty}\right)} = \frac{\left(\mp \frac{1}{\infty}\right)\left(2 \mp \frac{1}{\infty}\right)}{\left(\mp \frac{1}{\infty}\right)} = 2 \mp \frac{1}{\infty}$$

$$\Rightarrow \frac{x^2 - 1}{x - 1} \rightarrow 2 \Rightarrow \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2$$

Example 2.5: solve the following limit.

$$\lim_{x \rightarrow 0} \frac{1}{x^2 - 1}$$

Sol:

By lemma 1.12

$$x \rightarrow 0 \Rightarrow x = \mp \frac{1}{\infty}$$

$$\frac{1}{x^2 - 1} = \frac{1}{\left(\mp \frac{1}{\infty}\right)^2 - 1} = \frac{1}{\frac{1}{\infty^2} - 1} = \frac{1}{\frac{1}{\infty_1} - 1}$$

By remark 1.12

$$\infty_1 = \infty^2$$

$$\begin{aligned}
&= \frac{1}{\left(\frac{1 - \infty_1}{\infty_1}\right)} = \frac{\infty_1}{1 - \infty_1} = -\frac{\infty_1}{\infty_1 - 1} \\
&= -\frac{\infty_1 - 1 + 1}{\infty_1 - 1} = -\left(1 + \frac{1}{\infty_1 - 1}\right) \\
&= -\left(1 + \frac{1}{\infty_1 - 1}\right) = -\left(1 + \frac{1}{\infty_2}\right) = -1 - \frac{1}{\infty_2} \\
&\infty_2 = \infty_1 - 1 \\
&\Rightarrow \frac{1}{x^2 - 1} \rightarrow -1 \Rightarrow \lim_{x \rightarrow 0} \frac{1}{x^2 - 1} = -1
\end{aligned}$$

Example 2.6: find the following function limit.

$$\lim_{x \rightarrow 1} \frac{1}{x^2 - 1}$$

Sol:

By lemma 1.12

$$x \rightarrow 1 \Rightarrow x = 1 \mp \frac{1}{\infty}$$

$$\begin{aligned}
\frac{1}{x^2 - 1} &= \frac{1}{\left(1 \mp \frac{1}{\infty}\right)^2 - 1} = \frac{1}{1 \mp \frac{2}{\infty} + \frac{1}{\infty^2} - 1} \\
&= \frac{1}{\mp \frac{2}{\infty} + \frac{1}{\infty^2}} = \frac{1}{\frac{1}{\infty} \left(\mp 2 + \frac{1}{\infty}\right)} = \frac{\infty}{\mp 2 + \frac{1}{\infty}} = \mp \infty
\end{aligned}$$

The limit does not exist

Because the limit of the right is not equal to the limit of the left

The limit of the right

$$x \rightarrow a_+ \Leftrightarrow x = a + \frac{1}{\infty}$$

The limit of the left

$$x \rightarrow a_- \Leftrightarrow x = a - \frac{1}{\infty}$$

Example 2.7: solve the following limit.

$$\lim_{x \rightarrow 0} \frac{|x|}{x}$$

Sol:

The limit of the right, By lemma 1.12

$$x \rightarrow 0_+ \Leftrightarrow x = \frac{1}{\infty}$$

$$\frac{|x|}{x} = \frac{\left|\frac{1}{\infty}\right|}{\frac{1}{\infty}} = \frac{\left(\frac{1}{\infty}\right)}{\left(\frac{1}{\infty}\right)} = 1$$

The limit of the left, By lemma 1.12

$$x \rightarrow 0_- \Leftrightarrow x = \frac{-1}{\infty}$$

$$\frac{|x|}{x} = \frac{\left|\frac{-1}{\infty}\right|}{\frac{-1}{\infty}} = \frac{\left(\frac{1}{\infty}\right)}{\left(\frac{-1}{\infty}\right)} = -1$$

Right and left have different limits.

The limit is nonexistent.

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