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On Fuzzy Quasi Filter Topological R-Module Spaces

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Abstract. *This article deals with a new concept in the subject of fuzzy topological R-Module spaces is said fuzzy quasi Filter via fuzzy open quasi Filter sets and fuzzy closed quasi Filter sets and also present definitions for the Mixed fuzzy quasi Filter with this scope work and we briefly shows some features of fuzzy functions.*

Keywords: *fuzzy quasi Filter, fuzzy topological R-Modu., fuzzy nbhd quasi Filter, fuzzy open quasi Filter set, fuzzy closed quasi Filter set, Mixed fuzzy quasi Filter fuzzy topological R-Modu.*

1. INTRODUCTION AND PRELIMINARIES

The fuzzy set is a map $\mathbb{F}L: Q \rightarrow \{g: g \in [0,1]\}$ (Zadeh, 1965). the fzy Topological (fzy Topo.) space studied by (Chang, 1968). fzy Nbhd sys topo. R-Modu. showed by (Melgat, 2024a). Several types of fuzzy Filter are introduced by many authors (.Zhang, 2011; Amroune & Aissa, 2018; Chang, 1968; Eswarlal, 2015; Foster, 1979; Güloglu, 2010; Ibedou & Abbas, 2018; Jun et al., 2007; Jun & Ahn, 2017; Ma, 2015; Melgat, 2024a, 2024b; S.K.Samanta, 2011; Zhan et al., 2013, 2014; Zhang et al., 2014). Our gool is to given via fuzzy R-Modu. a new form of fzy R-Modu., we have devoted a class of fzy quasi Filter sets in a fzy R-Modu. to be a class of fzy open quasi Filter of fzy identity element 0 of the fuzzy quasi Filter Topological R-Modu.. (fzy quasi Filter Topo. R-Modu.). As an application, a mixed fuzzy quasi Filter Topological R-Modu. (Mix. fzy quasi Filter topo. R-Modu.) is formed from two given fzy quasi Filter topo. R-Modules with the help of fzy open quasi Filter and the presented the terms $\mathbb{FQ}K_1(\mathbb{FQ}K_2))$ interior quasi Filter and $\mathbb{FQ}K_1(\mathbb{FQ}K_2))$ fzy closure quasi Filter.

Defi. 1.1 (Foster, 1979)

let W be a left R-Modu., the set Q is said to be left fzy Topo. R-Modu. if:

(1) Q left fzy R-Modu.

(2) Q is a fzy Topo. gp on W and satisfies the following axiom:

The mapping $\varphi: R \times Q \longrightarrow Q$, $\varphi(h,g) = h.g$, $(h \in R \text{ and } g \in Q)$ is a fzy cont.

Thm. 1.2 (Foster, 1979)

Let $(\mathrm{Q}, \aleph_{FQ})$ be a fzy topo. R-Modu. and $g \in Q$.Then the maps $L_p: (\mathrm{R}, \aleph_{FR}) \times (\mathrm{Q}, \aleph_{FQ}) \rightarrow (\mathrm{Q}, \aleph_{FQ})$, $L(h,g) = h.g$ is fzy homeoms.

Defin. 1.3 (Das & Das, 2000)

If Q be an R-Modu. with two fzy topo. R-Modu. $\aleph_{(FQ)_1}$ and $\aleph_{(FQ)_2}$. Then $(Q, \aleph_{(FQ)_1}, \aleph_{(FQ)_2})$ is known a bi- fzy topo. R-Modu. space.

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Defin. 1.4 (Das & Das, 2000)

Let$(Q, \aleph_{(FQ)_1}, \aleph_{(FQ)_2})$ be a bi- fzy topo. R-Modu. The fzy Topo. $\aleph_{(FQ)_1}(\aleph_{(FQ)_2})$ on Q known by the collection $\{K \in I^Q : \exists H \in \aleph_{(FQ)_2} s.t\ cl_{\aleph_{(FQ)_1}}(H) \leq K\}$ of all fzy open nbhds of 0 ,then $(Q, \aleph_{(FQ)_1}(\aleph_{(FQ)_2}))$ be a mixed. fzy topo. R-Modu.

Defin.1.5 (Melgat, 2024b)

A fzy Filter $\mathbb{F}$ on $Q \neq 1_0$ is class of subsets of I^Q s.t

(i) If $F_1, F_2 \in \mathbb{F}\ implies\ F_1 \wedge F_2 \in \mathbb{F}$

(ii) If $F_1 \in \mathbb{F}\ and\ F_1 \leq F_2\ implies\ F_2 \in \mathbb{F}$

(iii) $1_0 \notin \mathbb{F}$.

Defin.1.6 [3**]**

A collection $\{\delta\} \neq 1_0$ of fzy subsets of $\mathrm{I^Q}$ is a fzy base for some fzy Filter with

(i) if $\delta_1, \delta_2 \in \delta$ then $\delta_3 \leq \delta_1 \wedge \delta_2$for some $\delta_3 \in \delta$

(ii) $\emptyset \notin \{\delta\}$

the collection $\mathbb{F} = \{\mathrm{F} \in \mathrm{I^Q} :\ \exists\ \mathrm{U} \in \{\delta\}\ \mathrm{s.t}\ \mathrm{U} \leq \mathrm{F}\}$ is a fzy Filter.

Thm 1.7 (Das & Das, 2000)

Let $(Q, \aleph_{(FQ)_1}(\aleph_{(FQ)_2}))$ be any bi-fzy topo. R-Modu... If $\aleph_{(FQ)_1} < \aleph_{(FQ)_2}$, then $\aleph_{(FQ)_2} < \aleph_{(FQ)_1}(\aleph_{(FQ)_2}) < \aleph_{(FQ)_1}$

Thm. 1.8 (Das & Das, 2000)

If $(Q, \aleph_{(FQ)_1})$and $(Q, \aleph_{(FQ)_2})$ are fzy topo. R-Modu. s.t $\aleph_{(FQ)_1} \leq \aleph_{(FQ)_2}$. Let $F\delta_1, F\delta_2$ be a system of fzy nbhds of 0. Then $F\delta_1(F\delta_2) = \{V \in I^Q : \exists U \in \aleph_{(FQ)_2}\ s.t$ $cl_{\aleph_{(FQ)_1}}(U) \leq V\}$ is a fzy fund. system of fzy nbhds of 0.

Def. 1.9 (Foster, 1979)

A fzy set $U, V \leq Q$ is called quasi coincident if $U(f) + V(f) > 1$

Theorem 1.10 (Melgat, 2024a)

Suppose $(Q, \aleph_{FQ})$ be a fuzzy topo. R-modu. and let $\{\delta_0\}$ be a founs. of fzynbhd of 0 such that $\delta(0) = \max\{Q(f)\},\ \forall f \in Q\}$ of fzy top-R-module, then the next are true

1) $\forall \delta_1 \in \{\delta_0\}\ \exists \delta_2 \in \{\delta_0\}$ s.t $-\delta_2 \leq \delta_1$.

2) $\forall \delta_1 \in \{\delta_0\}\ \exists \delta_2 \in \{\delta_{0\}}$ s.t $\delta_2 + \delta_2 \leq \delta_1$.

3) $\forall \delta_1 \in \{\delta_0\}$ and $\delta_2 \in \{\delta_0\}\ \exists \delta_3 \in \{\delta_0\}$ s.t $\delta_3 \leq \delta_1 \wedge \delta_2$.

4) $\forall \delta_1 \in \{\delta_0\}$ and $\delta_2 \in \{\delta_0\}\ \exists \delta_3 \in \{\delta_0\}$ and a fuzzynbhd W of 0 in R s.t $W.\delta_2 \leq \delta_1$.

5) $\forall \delta_1 \in \{\delta_0\}$ and $h \in R\ \exists \delta_2 \in \{\delta_0\}$ s.t $h.\delta_2 \leq \delta_1$..

6) $\forall \delta_1 \in \{\delta_0\}$ and $g \in Q\ \exists \delta_2$ fuzzynbhd of 0 s.t $\delta_2.g \leq \delta_1$

2. METHODOLOGY

The present work will be done, depending on math. definitions, constructs and proves to get the results.

Fzy quasi Filter topo. and fzy R-Modu. will used to define the fzy quasi Filter topo. R-Modu. and mix. fzy topo. R-Modu. The properties will be given, including the existence of.fzy open quasi Filter and fzy T_{F_0}quasi Filter topo. space.

The term of bi-fzy topo. R-Modu. will be gave, take care of characteristic of fzy quasi Filter topo. and fzy R-Modu. The mix. of fuzziness of it. will be introduced.

3. RESULTS

3.1 Fuzzy Quasi Filter topo. R-Modu.

In this section we submitted fzy quasi Filter $\mathbb{FQ}$ topo. R-Modu. and fzy open quasi Filter of fzy quasi Filter topo. R-Modu.

Def 3.3.1

Let $\mathbb{FQ}$ be a fzy filter on Q The fzy sets $\mathbb{FQ}\,\mathrm{K}_1, \mathbb{FQ}\,\mathrm{K}_2 \in \mathbb{FQ}$ is fzy quasi filter iff $\mathbb{FQ}\,\mathrm{K}_1(h) + \mathbb{FFQ}\mathrm{K}_2(h) > \max\{\mathbb{FQ}K(h),\ \forall h \in Q\}$. If it is not fzy quasi filter then $\mathbb{FQ}\,\mathrm{K}_1(h) + \mathbb{FQ}\,\mathrm{K}_2(h) \leq \max\{\mathbb{FQ}K(h),\ \forall h \in Q\}$.

Defin. 3.3.2

A fzy quasi Filter $\mathbb{FQ}$ on fzy R-Modu. Q is a class of fzy quasi filter subsets of I^Q with:

(i) If $\mathbb{FQ}\mathrm{K}_1, \mathbb{FQ}\mathrm{K}_2 \in \mathbb{FQ}\ then\ \mathbb{FQ}\mathrm{K}_1 \wedge \mathbb{FQ}\mathrm{K}_2 \in \mathbb{FQ}$

(ii) If $\mathbb{FQ}\mathrm{K}_1 \in \mathbb{FQ}\ \ and\ \mathbb{FQ}\mathrm{K}_1 \leq\ \mathbb{FQ}\mathrm{K}_2 then\ \mathbb{FQ}\mathrm{K}_2 \in \mathbb{FQ}$

(iii) $1_0 \notin \mathbb{FQ}$.

(iv) $\mathbb{F}L: R \times Q \longrightarrow Q$, $\mathbb{F}L(h,g) = h.g, (h \in R\ and\ \ g \in Q)$

then $(Q, \mathbb{FQ})$ be a. fzy quasi filter R-Modu.

Defin. 3.3.3

Let $(Q, \mathbb{FQ}\ )$ be a fzy quasi Filter R-Modu. space. A subset $\mathbb{FQ}\mathrm{K} \in I^Q$ is fzy open quasi filter R-Modu. (respect to $\mathbb{FQ}$) iff $\forall g \in \mathbb{FQ}\mathrm{K}$ and $\mathbb{FQ}H \in \mathbb{FQ}(g)$ then $\mathbb{FQ}H \leq \mathbb{FQ}K$ and $(\mathbb{FQ}K)^c$ is said fzy closed quasi filter R-Modu. set.

Defin. 3.3.4

A mapping $\mathbb{F}\boldsymbol{L}$ from a fzy quasi Filter R-Modu. space$(Q, \mathbb{FQ}_1)$ into fzy quasi Filter R-Modu. space $(Q, \mathbb{FQ}_2)$ is fzy quasi Filter cont. iff $\mathbb{FQ}K \in \mathbb{F}_2$ implies $\mathbb{F}\boldsymbol{L}^{-1}(\mathbb{FQ}K) \in \mathbb{FQ}_1$.

Thm. 3.3.5

Let $(Q, \mathbb{FQ})$ be a fzy quasi Filter R-Modu. and $g \in Q$. Then $\mathbb{F}L: R \times Q \longrightarrow Q$, $\mathbb{F}L(h,g) = h.g, (h \in R \; and \; g \in Q)$ is fzy homeoms.

Proof

Clearly $\mathbb{F}L$ is $1-1$.

Since $\mathbb{F}L(h,g) = Q(h.g) = Q(f)$ for all $h \in R$ and $f, g \in Q$, $\mathbb{F}L(R \times Q) = Q$implies $\varphi$is onto

And Since $\mathbb{F}L^{-1}(h.g) = (h,g)$ is fzy quasi Filter cont. Thus, $\mathbb{F}L$ is a fzy quasi Filter homeom.

Thm. 3.3.6

If $(Q, \mathbb{FQ})$ be a fzy quasi Filter R-Modu. space and let $\{\mathbb{FQ}K_0\} \leq I^Q$ be a fzy open quasi Filter (with respect to $\mathbb{FQ}$) then $(Q, \aleph_{\mathbb{FQ}})$ is a fzy topo. R-Modu. on Q induced by $\mathbb{FQ}$,where $\aleph_{\mathbb{FQ}} = 1_0 \vee \{\mathbb{FQ}K_0\}$.

Proof : Clearly

Theorem 3.3.7

Suppose $(Q, \aleph_{\mathbb{FQ}})$ is be a fuzzy top-R-module and let $\{\mathbb{FQ}K_0\}$ be a fundamental of fzy open quasi filter of 0 such that $\mathbb{FQ}K(0) = \max\{Q(g)\}, \forall g \in Q\}$ of fz-top R-module, then the following statement are true

1) $g \in \wedge(\mathbb{FQ}K + \mathbb{FQ}H)$

2) for any $\mathbb{FQ}K \in \{\mathbb{FQ}K_0\}$ and $\mathbb{FQ}H \in \{\mathbb{FQ}K_0\}$there exist $\mathbb{FQ}W \in \{\mathbb{FQ}K_0\}$ s.t $\mathbb{FQ}W \leq \mathbb{FQ}K \cap \mathbb{FQ}H$.

3) for any $\mathbb{FQ}K \in \{\mathbb{FQ}K_0\}$ there exist $\mathbb{FQ}H \in \{\mathbb{FQ}K_0\}$s.t $\mathbb{FQ}H + \mathbb{FQ}H \leq \mathbb{FQ}K$.

4)for any $\mathbb{FQ}K \in \{\mathbb{FQ}K_0\}$ there exist $\mathbb{FQ}H \in \{\mathbb{FQ}K_0\}$s.t $\mathbb{FQ}H \leq \mathbb{FQ}H$.

5) $\forall\, \mathbb{FQ}K \in \{\mathbb{FQ}K_0\}$ t$\exists$ $\mathbb{FQ}H \in \{\mathbb{FQ}K_0\}$and a fzy quasi filter $\mathbb{FQ}W$ of 0 in R s.t $\mathbb{FQ}W . \mathbb{FQ}H \leq \mathbb{FQ}K$.

6) for any $\mathbb{FQ}K \in \{\mathbb{FQ}K_0\}$ and $h \in R$ there exist $\mathbb{FQ}H \in \{\mathbb{FQ}K_0\}$ s.t $h \,.\, \mathbb{FQ}H \leq \mathbb{FQ}K$.

7) for any $\mathbb{FQ}K \in \{\mathbb{FQ}K_0\}$ and $g \in Q$ there exist a fzy quasi filter $\mathbb{FQ}H$ of 0 in R s.t $\mathbb{FQ}H \,.\, g \leq \mathbb{FQ}K$.

Proof

1,2,3,4 and 5 By theorem 1.10 and definition 3.3.2

(6) Let $\mathbb{FQ}K$ be an fzy quasi filter of 0 and $\mathbb{FQ}K(0) > 0$, then $\exists \mathbb{FQ}H \in \aleph_{\mathbb{FQ}}$,$\mathbb{FQ}H\,(0) > 0$ s.t $\mathbb{FQ}H \leq \mathbb{FQ}K$ and let $\mathbb{F}L: R \times Q \rightarrow Q$; $\mathbb{F}L(h,g) = h.g$, then by thm3.3.5 is fuzzy homeo. therefore $h \,.\, \mathbb{FQ}H$ is fzy open quasi filter of 0.

(7) Let $\mathbb{FQ}K$ be an fzy quasi filter of 0 and $\mathbb{FQ}K(0) > 0$, then $\exists \mathbb{FQ}H \in \aleph_{\mathbb{FQ}}$, $\mathbb{FQ}H\ (0) > 0$ s.t $\mathbb{FQ}H \leq \mathbb{FQ}K$ and let $\mathbb{F}L: R \times Q \to Q$; $\mathbb{F}L(h, g) = h.g$, then by thm 3.3.5 is fuzzy homeo. therefore $\mathbb{FQ}H\ .\ g$ is fzy open quasi filter of 0.

Corollary 3.3.8

Let $\{\mathbb{FQ}\mathrm{W}_0\}$ be a fzy quasi Filter base on Q then,

1) for any $\mathbb{FQ}K \in \{\mathbb{FQ}\mathrm{W}_0\}$ and $\mathbb{FQ}H \in \{\mathbb{FQ}\mathrm{W}_0\}$there exist $\mathbb{FQ}W \in \{\{\mathbb{FQ}\mathrm{W}_0\}$s.t $\mathbb{FQ}W \leq \mathbb{FQ}K \cap \mathbb{FQ}H$.

2) for any $\mathbb{FQ}K \in \{\mathbb{FQ}W_0\}$ there exist $\mathbb{FQ}H \in \{\mathbb{FQ}W_0\}$s.t $\mathbb{FQ}H + \mathbb{FQ}H \leq \mathbb{FQ}K$.

3)for any $\mathbb{FQ}K \in \{\mathbb{FQ}W_0\}$ there exist $\mathbb{FQ}H \in \{\mathbb{FQ}W_0\}$s.t $\mathbb{FQ}H \leq \mathbb{FQ}H$.

4) $\forall\ \mathbb{FQ}K \in \{\mathbb{FQ}W_0\}$ t$\exists$ $\mathbb{FQ}H \in \{\mathbb{FQ}W_0\}$and a fzy quasi filter $\mathbb{FQ}W$ of 0 in R s.t $\mathbb{FQ}W.\mathbb{FQ}H \leq \mathbb{FQ}K$.

5) for any $\mathbb{FQ}K \in \{\mathbb{FQ}W_0\}$ and $h \in R$ there exist $\mathbb{FQ}H \in \{\mathbb{FQ}W_0\}$ s.t $h\ .\ \mathbb{FQ}H \leq \mathbb{FQ}K$.

6) for any $\mathbb{FQ}K \in \{\{\mathbb{FQ}\mathrm{W}_0\}\}$ and $g \in Q$ there exist a fzy quasi filter $\mathbb{FQ}H$ of 0 in R s.t $\mathbb{FQ}H\ .\ g \leq \mathbb{FQ}K$.

Conversely, there is only fzy topo. R-Modu. on Q for $\{\mathbb{FQ}\mathrm{W}_0\}$ is a fzy open quasi Filter base of 0

Thm 3.3.9

Let Q be a fzy R-Modu and $g \in Q$ be an invertible fzy pt s.t $g \in \{f : \mathbb{FQ}K(f) = \max\{\mathbb{FQ}K(h)\},\ \forall h \in Q\}$. Let $\{\mathbb{FQ}K_0\}$ be a fundamental fzy open quasi Filter of 0 justifzying (1) – (6) of Thm (3.3.7), there is only fzy topo. R-Modu.. $\aleph_{\mathbb{FQ}}$ is on Q where $\aleph_{\mathbb{FQ}} = 1_0 \vee \{\mathbb{FQ}K_0\}$.

Proof

Clearly that $(Q, \aleph_{\mathbb{FQ}})$ is a unique fuzzy topo. R-Modu... We claim the following map $\mathbb{F}L: (\mathrm{R}, \aleph_{\mathbb{FQ}}) \times (\mathrm{Q}, \aleph_{\mathbb{FQ}}) \longrightarrow (\mathrm{Q}, \aleph_{\mathbb{FQ}})$, $\mathbb{F}L(h, g) = h.g$, $h \in R$, $g \in Q$, is a fuzzy Contin.

Now let $\mathbb{FQ}K$ be a fzy open quasi Filter of $h.g$, then $\mathbb{F}L^{-1}(\ \mathbb{FQ}K)(h.g) = \mathbb{FQ}K\big(\mathbb{F}L(h, g)\big) = \mathbb{F}K(h.g) > 0$. by (Thm 3.3.7-5) there exists a fzy open quasi Filter in R $\mathbb{FQ}W.$ of h s.t $\mathbb{F}L(\mathbb{FQ}W \times \mathbb{FQ}H) = \mathbb{FQ}W.\mathbb{FQ}H \leq \mathbb{FQ}K$

Thus $\mathbb{F}L$ is fzy Contin

Thm 3.3.10

Let$(Q, \aleph_{\mathbb{FQ}})$ fuzzy topo. R-Modu and $\{\mathbb{FQ}K_0\}$ fzy open quasi Filter of 0, A fzy topo. R-Modu. $(\mathrm{Q}, \varepsilon_{F_Q})$ is fzy T_{F_0} –topo. R-Modu. space iff $\{0\}$ is fzy closed quasi Filter set

Proof: Clearly

Corollary 3.3.11

Let [$\mathbb{FQ}$W} be a fzy quasi Filter base of 0 then $(Q, \aleph_{\mathbb{FQ}})$ is fzy T_{F_0} −topo. R-Modu... space iff {0} is fzy closed quasi Filter.

4. MIXED FUZZY QUASI FILTER TOPOLOGICAL R-MODU.

Defin. 4.1

Let $(Q, \aleph_{\mathbb{FQ}_1}), (Q, \aleph_{\mathbb{FQ}_2})$ be fzy R-Modu. Then the triple $(Q, \aleph_{\mathbb{FQ}\,1}, \aleph_{\mathbb{FQ}\,2})$ is known a bi- fzy quasi Filter R-Modu. space.

Defin. 4.2

Let$(Q, \aleph_{\mathbb{FQ}\,1}, \aleph_{\mathbb{FQ}\,2})$ be a bi- fzy quasi Filter R-Modu. The fzy quasi Filter R-Modu. $\aleph_{\mathbb{FQ}\,1}(\aleph_{\mathbb{FQ}\,2})$ on Q by the collection $\{\mathbb{FQ}K \in I^Q : \exists \mathbb{FQ}H \in \aleph_{\mathbb{FQ}\,2}\ s.t\ cl_{\aleph_{\mathbb{FQ}\,1}}(\mathbb{FQ}H) \leq \mathbb{FQ}K\}$ of all fzy open quasi Filter of 0 s.t $(Q, \aleph_{\mathbb{FQ}\,1}(\aleph_{\mathbb{FQ}\,2}))$ be a fzy quasi Filter. R-Modu. is known as a mixed fzy quasi Filter R-Modu...

Theorem 3.3.2

Let $(Q, \aleph_{(\mathbb{FQ})_1})$and $(Q, \aleph_{(\mathbb{FQ})_2})$ be two fzy tops. R-Modu such that $\aleph_{(\mathbb{FQ})_1} \leq \aleph_{(\mathbb{FQ})_2}$. Let δ_1, δ_2 be a fund. fzy quasi filter of identity $0 \in Q$ in the fuzzy top. spaces $\aleph_{(\mathbb{FQ})_1}$, $\aleph_{(\mathbb{FQ})_2}$ respectively. Then

$\delta_1(\delta_2) = \{\mathbb{FQ}U \in I^Q : \exists \mathbb{FQ}V \in \aleph_{(FQ)_2}\ s.t\ cl_{\aleph_{(FQ)_1}}(\mathbb{FQ}V) \leq \mathbb{FQ}U\}$ is a fundamental fzy quasi filter of 0

Proof

We want to show that the conditions of Theorem 3.3.7 are justified by $\delta_1(\delta_2)$.

(**1**) Let $\mathbb{FQ}U \in \delta_1(\delta_2)$, then there is a fzy open quasi filter $\mathbb{FQ}V \in \delta_2$ s.t $cl_{\aleph_{\mathbb{FQ}\,1}}(\mathbb{FQ}V) \leq \mathbb{FQ}U$, by condition 1 theorem 3.3.7, $-\mathbb{FQ}V \in \delta_2$ Now $cl_\beta(-\mathbb{FQ}V) = -cl_\beta(\mathbb{FQ}V) \leq -\mathbb{FQ}$U. Also $-\mathbb{FQ}U(0) = \mathbb{FQ}U(-0) = \mathbb{FQ}U(0) > 0$

Thus

$$-\mathbb{FQ}U \in \delta_1(\delta_2)$$

(**2**) Let $\mathbb{FQ}U \in \delta_1(\delta_2)$Then ∃ fzy open quasi filter $\mathbb{FQ}V \in \delta_2$ by condition 2 theorem3.3.7, $\mathbb{FQ}V + \mathbb{FQ}V \in \delta_2$. Now,

$$cl_{\aleph_{\mathbb{FQ}\,1}}(\mathbb{FQ}V) + cl_{\aleph_{\mathbb{FQ}\,1}}(V) \subseteq \mathbb{FQ}U + \mathbb{FQ}U$$

Also $(\mathbb{FQ}U + \mathbb{FQ}U)(0) = supmin\{\mathbb{FQ}U(0), \mathbb{FQ}U(0)\} = \mathbb{FQ}U(0) > 0$

Thus, $\mathbb{FQ}U + \mathbb{FQ}U \in \delta_1(\delta_2)$

(**3**) Let $\mathbb{FQ}U_1, \mathbb{FQ}U_2 \in \delta_1(\delta_2)$then there is a fzy open quasi filter$\mathbb{FQ}V_1, \mathbb{FQ}V_2 \in \delta_2$ s.t $cl_\beta(\mathbb{FQ}V_1) \leq \mathbb{FQ}U_1$ and $cl_\beta(\mathbb{FQ}V_2) \leq \mathbb{FQ}U_2$. Since $\mathbb{FQ}V_1, \mathbb{FQ}V_2 \in \delta_2$ then by condition 3 theorem 3.3.7, we have $\mathbb{FQ}V_1 \wedge \mathbb{FQ}V_2 \in \delta_2$.

Now

$$cl_\beta(\mathbb{FQ}V_1 \wedge \mathbb{FQ}V_2) \leq cl_{\aleph_{\mathbb{FQ}\,1}}(\mathbb{FQ}V_1) \wedge cl_{\aleph_{\mathbb{FQ}\,1}}(\mathbb{FQ}V_2) \leq \mathbb{FQ}U_1 \wedge \mathbb{FQ}U_2$$

Thus,

$\mathbb{FQ}U_1 \wedge \mathbb{FQ}U_2 \in \delta_1(\delta_2)$. Also $(\mathbb{FQ}U_1 \wedge \mathbb{FQ}U_2)(0) = \min\{\mathbb{FQ}U_1(0), \mathbb{FQ}U_2(0)\} > 0$

(**4**) Let $\mathbb{FQ}U \in \delta_1(\delta_2)$then there exists $\mathbb{FQ}V \in \delta_2$ and $\mathbb{FQ}W \in R$ s.t $cl_{\aleph_{\mathbb{FQ}\,1}}(\mathbb{FQ}V) \leq \mathbb{FQ}U$, then by condition 4 theorem3.3.7, we have $\mathbb{FQ}W.\mathbb{FQ}V \leq \mathbb{FQ}U$.

Now

$$cl_\beta(\mathbb{FQ}V_1.\mathbb{FQ}V_2) \leq cl_{\aleph_{\mathbb{FQ}\,1}}(\mathbb{FQ}V_1)\,.\,cl_{\aleph_{\mathbb{FQ}\,1}}(\mathbb{FQ}V_2) \leq \mathbb{FQ}U_1\,.\,\mathbb{FQ}U_2$$

$$\mathbb{FQ}W.\mathbb{FQ}V(0) = \min\{\mathbb{FQ}W(0), \mathbb{FQ}V(0)\} > 0$$

Thus, $\mathbb{FQ}W.\mathbb{FQ}V \in \delta_1(\delta_2)$

(**5**)

By the same way of (4) we can prove that $h.\mathbb{FQ}U \in \delta_1(\delta_2)$

(**6**)

Let $\mathbb{FQ}U \in \delta_1(\delta_2)$, $g \in Q$ and $\mathbb{FQ}V$ a fzy quasi filter of 0 in R, then by condition 6 of theorem 3.3.7 we get $\mathbb{FQ}V.h \leq \mathbb{FQ}U$. Now, $cl_\beta(\mathbb{FQ}V.h) = cl_{\aleph_{\mathbb{FQ}\,1}}(\mathbb{FQ}V).h \leq \mathbb{FQ}U.h$. Also $(\mathbb{FQ}U.h)(0) = \mathbb{FQ}U(0) > 0$. Thus, $\mathbb{FQ}U.h \in \delta_1(\delta_2)$

Coro. 4.4

Let $(\boldsymbol{Q}, \mathbb{F}_{\mathbb{Q}1})$ and$(Q, \mathbb{FQ}_2)$ be two fzy quasi Filter R-Modu.. such that $\mathbb{FQ}_1, \mathbb{FQ}_2$. Let $\{\mathbb{FQ}W_1\}, \{\mathbb{FQ}W_2\}$ is fzy open quasi Filter base of 0 in the $\mathbb{FQ}_1, \mathbb{FQ}_2$ respectively. Then $\{\mathbb{FQ}W_1(\mathbb{FQ}W_2)\} = \{\mathbb{FQ}K \in I^Q : \exists \mathbb{FQ}H \in \mathbb{FQ}_2 \; s.t \; cl_{\mathbb{F}_1}(\mathbb{FQ}H) \leq \mathbb{FQ}K\}$ is a fzy open quasi Filter basis of 0. Conversely, there is only fzy topo. R-Modu.. on Q for $\{\mathbb{FQ}W_1(\mathbb{FQ}W_2)\}$.

Thm 4.5

Let$\{\mathbb{FQ}K_1(\mathbb{FQ}K_2)\} = \{\mathbb{FQ}K \in I^Q : \exists \mathbb{FQ}H \in \mathbb{FQ}_2 \; s.t \; cl_{\mathbb{FQ}_1}(\mathbb{FQ}H) \leq \mathbb{FQ}K\}$ be a fund. of fuzzy open quasi Filter of 0 satisfied the conditions of Thm 3.3.3. then ∃! fzy topoo. R-Modu.. $\aleph_{\mathbb{FQ}\,1}(\aleph_{\mathbb{FQ}\,2})$ s.t $(Q, \aleph_{\mathbb{FQ}\,1}(\aleph_{\mathbb{FQ}\,2}))$ is mix. fzy topoo. R-Modu. where $\aleph_{\mathbb{FQ}\,1}(\aleph_{\mathbb{FQ}\,2}) = 1_0 \vee \{\mathbb{FQ}K_1(\mathbb{FQ}K_2)\}$

Proof: By the same way of Thm 3.3.9

5. DISCUSSION

This paper first introduces the concept "fzy quasi Filter topo. R-Modu..", These studies give us a link between fzy quasi Filter topo. and fzy R-Modu.. theory which is the combined form of fzy quasi Filter topo. and fzy R-Modu... This paper reports on the more generalized form of fuzzy topo. R-Modu.. definition that includes bi fzy quasi Filter topo. spaces as a special case. The properties of fzy quasi Filter topo. R-Modu.., fuzzy open quasi Filter , fuzzy closed quasi Filter , fzy quasi Filter base and mix. fzy quasi Filter topo.R-Modu.. are analyzing and consistent with the earlier findings on fzy quasi Filter topo. spaces and fzy algebraic structures too. The work constructs mix. Fzy quasi Filter topo. R-Modu.. by using fzy open quasi Filter to bi-fzy quasi Filter topo. R-Modu... This work is similar to the mix. fzy topo. R-Modu.. spaces gave by (Das & Das, 2000). In Das's construction, two fzy R-Modu.. with distinct fzy topos. are put together to create a new mix. fzy topo. R-Modu.. space.

If Q be a set and $(\mathrm{Q}, \mathbb{F}_1, \mathbb{F}_2)$ be a bi- fzy quasi Filter R-Modu.. Then fzy quasi Filter topo. R-Modu.. $\varepsilon_{(FQ)_1}(\varepsilon_{(FQ)_2})$ on Q indicated in this article finding $\aleph_{\mathbb{FQ}_1}(\aleph_{\mathbb{FQ}_2})$is a mix. fzy quasi Filter topo. R-Modu... structure also we obtain that for any mix. fzy quasi Filter on fzy R-Modu.. is a fzy quasi Filter topo. R-Modu. In future this work can also be extended further on Bolian algebra and also in other areas.

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