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Neutrosophic AT-ideals of AT-algebra with Applications

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Abstract*. This work introduces the concept of neutrosophic AT-ideals in AT-algebra and presents some associated facts and theorems. The homomorphism of neutrosophic AT-ideals in AT-algebra is examined, and some findings pertaining to them under homomorphism are derived.*

Keywords: *AT-algebra, neutrosophic AT-ideal, neutrosophic AT-algebra. AMS Classification. 06F35.*

1. INTRODUCTION

A BCK-algebra is a significant category of logical algebras established by C. Is´eki K. and S. Tanaka, and has been thoroughly examined by numerous academics. The category of all KU-algebras is qualitative. They introduced the concept of homomorphism in KU-algebras and examined several associated properties. The notion of fuzzy subsets and their associated operations were initially proposed by L.A. Zadeh, after which fuzzy subsets have been utilized across numerous domains. Fuzzy algebra constitutes a significant domain within fuzzy mathematics. The concept of fuzzy subgroups was established in 1971 by A. Rosenfeld. These concepts have been implemented in several algebraic structures, including semigroups, rings, ideals, modules, and vector spaces. O.G. Xi adapted this concept to BCK-algebra and created the notion of fuzzy subalgebras (ideals) of BCK-algebras concerning the minimum. S.M. Mostafa et al. proposed the concepts of KUS-algebras, KUS-ideals, and KUS-subalgebras, and examined the relationships among these entities. A.T. Hameed and et al have introduced the notion of some types of algebras as (AT-algebras, AB-algebras, QS-algebras SA-algebras) and gave the concept of ideals, subalgebra of them and investigates the relations among them. A.T. Hameed et al. have proposed the concept of fuzzy ideals and fuzzy subalgebras within KK-algebras, examined the interrelations among these constructs, and defined the notion of homomorphism of KK-algebras while exploring other associated features. A.T. Hameed et al. have presented fuzzy RG-ideals of RG-algebras, and we subsequently examine various fundamental features associated with fuzzy RG-ideals. This document elucidates the handling of homomorphisms concerning the image as well as inverse image of fuzzy RG-ideals. Neutrosophy is a novel philosophical discipline that examines the nature, origin, and extent of neutrals, along with their interactions across many intellectual domains. Smarandache

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introduced neutrosophic set and neutrosophic logic in 1998 as extensions of fuzzy set as well as intuitionistic fuzzy logic. Kandasamy and Smarandache established the concept of neutrosophic algebraic structures in 2006. Agboola and Davvaz presented the notion of neutrosophic BCI/BCK-algebras. Consequently, other researchers have investigated the themes, yielding a significant corpus of literature. This work presents a neutrosophic AT-ideal of an AT-algebra, examines its associated features, and explores a homomorphism of neutrosophic structures within an AT-algebra.

2. PRELIMINARIES

We will now review established notions pertaining to AT-algebra from the literature that will aid in the further examination of this subject.

Definition 2.1. ([5]). Suppose that $(\chi; *, 0)$ represent an algebra of type (2,0) characterized by a singular binary operation $(*)$. χ is designated as **an AT-algebra** (AT-A) if it fulfills the subsequent identities: for each $\varepsilon, \rho, \sigma \in \chi$,

(AT_1) : $(\varepsilon * \rho) * ((\rho * \sigma) * (\varepsilon * \sigma)) = 0$,

(AT_2) : $0 * \varepsilon = \varepsilon$,

(AT_3) : $\varepsilon * 0 = 0$.

In $(\chi; *, 0)$, a binary relation $(\leq)$ can be defined such that $\varepsilon \leq \rho$ if and only if $\rho * \varepsilon = 0$.

Lemma 2.2. ([5]). In every AT-A$(\chi; *, 0)$, the subsequent properties are valid for any $\varepsilon, \rho, \sigma \in \chi$

a) $\rho * \rho = 0$.

b) $\rho * ((\rho * \varepsilon) * \varepsilon) = 0$.

c) $\varepsilon \leq \rho$ implies that $\rho * \sigma \leq \varepsilon * \sigma$,

d) $\varepsilon \leq \rho$ implies that $\sigma * \varepsilon \leq \sigma * \rho$,

e) $\varepsilon * \rho \leq \sigma$ imply $\sigma * \rho \leq \varepsilon$

Example 2.3. ([5]).

1- Given $\chi = \{0, 1, 2, 3\}$, where (*) is delineated by the subsequent table:

table 1. delineated by the subsequent

$*$	0	1	2	3
0	0	1	2	3
1	0	0	3	2
2	0	2	0	1
3	0	3	1	0

Then $(\chi; *, 0)$, is AT-A.

2- Consider $\chi = \{0, 1, 2, 3, 4\}$, where the operation * is delineated by the subsequent table.

table 2. delineated by the subsequent

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	3
3	0	0	2	0	2
4	0	0	0	0	0

It is straightforward to demonstrate that $(\chi; *, 0)$, is AT-A.

Definition 2.4. ([5]). Letting $(\chi; *, 0)$ be an algebraic structure AT-A, and let S be a nonempty subset of χ. S is designated as **an AT-subalgebra of χ (AT-S)** if $\varepsilon * \rho$ belongs to S whenever ε and ρ are elements of S.

Definition 2.5. ([5]). A nonempty subset H of AT-A$(\chi; *, 0)$, is called **an ideal of χ** if it satisfies: for $\varepsilon, \rho \in \chi$

(I_1) $0 \in H$,

(I_2) $\varepsilon * \rho \in H$ and $\varepsilon \in H$ imply $\rho \in H$.

Proposition 2.6. ([5]). Every ideal of AT-A is a AT-S.

Definition 2.7. ([5]). A nonempty subset H of AT-A$(\chi; *, 0)$, is called **an AT-ideal of χ (AT-I)** if it satisfies: for $\varepsilon, \rho, \sigma \in \chi$

(ATI_1) $0 \in H$,

(ATI_2) $\varepsilon * (\rho * \sigma) \in H$ and $\rho \in H$ imply $\varepsilon * \sigma \in H$.

Proposition 2.8. ([5]). Every AT-I of AT-A is an ideal.

Proposition 2.9. ([5]). Every AT-I of AT-A is a AT-S.

Definition 2.10. ([5]). Let $(\chi; *, 0)$, be a AT-A, then for every $\varepsilon, \rho \in \chi$, we denote $\varepsilon \wedge \rho = (\varepsilon * \rho) * \rho$.

3. NEUTROSOPHIC AT-ALGEBRA

In neutrosophic logic, every assertion has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), with T, I, and F being standard or non-standard subsets of [-0,1+].

Definition 3.1. Let χ represent a nonempty set as well as letting I denote an indeterminate. The set

$\chi(I) = <\chi, I> = \{(\varepsilon, \rho I): \varepsilon, \rho \in \chi\}$ is referred to as a neutrosophic set produced by χ and I.

If $\bullet$ and $\diamond$ are standard, then I possesses the following attributes:

1) $I \bullet I \bullet \ldots \bullet I = nI$,

2) $I \bullet (-I) = 0$,

3) $I \diamond I \diamond \ldots \diamond I = I^n = I$, for every positive integer n

4) $0 \diamond I = 0$,

5) $-I$, is undefined and hence does not exist.

Remark 3.2.

a) If $\Upsilon: \chi(I)\times\chi(I) \rightarrow \chi(I)$ is a binary operation defined on $\chi(I)$, then the pair $(\chi(I), \Upsilon)$ is designated as a neutrosophic algebraic structure, named in accordance with the axioms fulfilled by Υ.

b) Consider $(\chi(I), \Upsilon)$ and $(\tau(I), \Upsilon')$ be two neutrosophic algebraic structures. A mapping f : $(\chi(I), \Upsilon) \rightarrow (\tau(I), \Upsilon')$ is termed a neutrosophic homomorphism if the subsequent criteria are satisfied.

(1) $f\{(\nu, \varepsilon I)\Upsilon(\rho, \sigma I)\} = f(\nu, \varepsilon I)\Upsilon' f(\rho, \sigma I)$,

(2) $f(I) = I$, for all $(\nu, \varepsilon I), (\rho, \sigma I) \in \chi(I)$.

Definition 3.3. Let $(\chi; *, 0)$ be any AT-algebra and $\chi(I) =< \chi, I >$ be a set generated by χ and I. The triple $(\chi(I), \Upsilon, (0,0))$ is called **a neutrosophic AT-algebra**, if $(\nu, \varepsilon I)$ and $(\rho, \sigma I)$ are any two elements of $\chi(I)$ with $\nu, \varepsilon, \rho, \sigma \in \chi$, we define the following

$(\nu, \varepsilon I)\Upsilon(\rho, \sigma I) = (\nu\Upsilon\rho, (\nu\Upsilon\sigma \wedge \varepsilon\Upsilon\rho \wedge \varepsilon\Upsilon\sigma)I)$….(1).

An element $\varepsilon \in \chi$ is denoted by $(\varepsilon, 0) \in \chi(I)$, while (0,0) signifies the constant element in $\chi(I)$.

For every $(\varepsilon, 0), (\rho, 0) \in \chi$, we get $(\varepsilon, 0)\,\Upsilon(\rho, 0) = (\varepsilon\Upsilon\rho, 0) = \rho \wedge \neg\varepsilon$ …..(2), where $\neg\varepsilon$ is the negation of $\varepsilon \in \chi$.

Proposition 3.4. Every neutrosophic AT-algebra $(\chi(I), \Upsilon, (0,0))$ with condition $(0,0I)\Upsilon(\rho, \sigma I) = (\rho, (\rho \wedge \sigma)I)$ is an AT-algebra.

Proof. Letting $(\chi(I), \Upsilon, (0,0))$ represent a neutrosophic AT-algebra. Permit $w = (\varepsilon, \rho I)$, $r = (\sigma, \varrho I)$ and $t = (\tau, \iota I)$ be an arbitrary elements of $\chi(I)$. Then we have (AT_1),

$(w * r) * ((r * t) * (w * t))$

$= ((\varepsilon, \rho I)\Upsilon(\sigma, \varrho I))\Upsilon(((\sigma, \varrho I)\Upsilon(\tau, \iota I))\Upsilon((\varepsilon, \rho I)\Upsilon(\tau, \iota I)))$

$= (\varepsilon\Upsilon\sigma, (\varepsilon\Upsilon\varrho \wedge \rho\Upsilon\sigma \wedge \rho\Upsilon\varrho)I)\Upsilon((\sigma\Upsilon\tau, (\sigma\Upsilon\iota \wedge \varrho\Upsilon\tau \wedge \varrho\Upsilon\iota)I)\Upsilon(\varepsilon\Upsilon\tau, (\varepsilon\Upsilon\iota \wedge \rho\Upsilon\tau \wedge \rho\Upsilon\iota)I))$

$= ((k, sI)\Upsilon((p, qI)\Upsilon(u, zI))$

$(k, sI) = (\varepsilon\Upsilon\sigma, (\varepsilon\Upsilon\varrho \wedge \rho\Upsilon\sigma \wedge \rho\Upsilon\varrho)I)$

$(p, qI) = (\sigma\Upsilon\tau, (\sigma\Upsilon\iota \wedge \varrho\Upsilon\tau \wedge \varrho\Upsilon\iota)I)$

$(u, zI) = (\varepsilon \curlyvee \tau, (\varepsilon \curlyvee \iota \wedge \rho \curlyvee \tau \wedge \rho \curlyvee \iota)I)$.

Hence,

$(p, qI) \curlyvee (u, zI) = (p \curlyvee u, (p \curlyvee z \wedge q \curlyvee u \wedge q \curlyvee z)I)$

$= (u \wedge \neg p, (z \wedge \neg p \wedge u \wedge \neg q \wedge z \wedge \neg q)I) = (m, nI)$

$(k, sI) \curlyvee (m, nI) = (k \curlyvee m, (k \curlyvee n \wedge s \curlyvee m \wedge s \curlyvee n)I)$

$= (m \wedge \neg k, (n \wedge \neg k \wedge m \wedge \neg s \wedge n \wedge \neg s)I) = (g, hI)$.

Now, we obtain $g = m \wedge \neg k = u \wedge \neg p \wedge \neg k = (\tau \wedge \neg \varepsilon)(\neg \sigma \vee \varepsilon) = 0$ and $h = n \wedge \neg k = m \wedge \neg s \wedge n \wedge \neg s = n \wedge \neg k \wedge u \wedge \neg p \wedge \neg s \wedge n \wedge \neg s = 0$. This shows that $(g, hI) = (0, 0I)$ and consequently$(w * r) * ((r * t) * (w * t)) = 0$.

$\mathbf{0} * \boldsymbol{w} = (\mathbf{0}, \mathbf{0}\boldsymbol{I}) \curlyvee (\boldsymbol{\varepsilon}, \boldsymbol{\rho I}) = (\mathbf{0} \curlyvee \boldsymbol{\varepsilon}, (\mathbf{0} \curlyvee \boldsymbol{\rho} \wedge \mathbf{0} \curlyvee \boldsymbol{\varepsilon})\boldsymbol{I}) = (\boldsymbol{\varepsilon}, (\boldsymbol{\rho} \wedge \boldsymbol{\varepsilon})\boldsymbol{I}) = (\boldsymbol{\varepsilon}, \boldsymbol{\rho I}) = \boldsymbol{w}$.

$\boldsymbol{w} * \mathbf{0} = (\boldsymbol{\varepsilon}, \boldsymbol{\rho I}) \curlyvee (\mathbf{0}, \mathbf{0}\boldsymbol{I}) = (\boldsymbol{\varepsilon} \curlyvee \mathbf{0}, (\boldsymbol{\rho} \curlyvee \mathbf{0} \wedge \boldsymbol{\varepsilon} \curlyvee \mathbf{0})\boldsymbol{I}) = (\mathbf{0}, (\mathbf{0} \wedge \mathbf{0})\boldsymbol{I}) = (\mathbf{0}, \mathbf{0}\boldsymbol{I})$.

Therefore, $(\boldsymbol{\chi}(\boldsymbol{I}), \boldsymbol{\curlyvee}, (\mathbf{0}, \mathbf{0}))$ is an AT-algebra.

Definition 3.5. Letting $(\chi(I), \curlyvee, (0,0))$ represent a neutrosophic AT-algebra, and let $S(I)$ be a nonempty of $\chi(I)$. $S(I)$ is called **an neutrosophic AT-subalgebra of $\boldsymbol{\chi}$ (NAT-S)** if

1) $(0,0) \in S(I)$

2) $(\varepsilon, \rho I) \curlyvee (\sigma, \varrho I) \in S(I)$ whenever $(\varepsilon, \rho I), (\sigma, \varrho I) \in S(I)$,

3) $S(I)$ contains a proper subset which is an AT-algebra.

Definition 3.6. Let $(\chi(I), \curlyvee, (0,0))$ be neutrosophic AT-algebra and let $H(I)$ be a nonempty of $\chi(I)$. $H(I)$ is called **an neutrosophic ideal of $\boldsymbol{\chi}$ (NI)** if

1) $(0,0) \in H(I)$

2) $(\varepsilon, \rho I) \curlyvee (\sigma, \varrho I) \in H(I)$ and $(\varepsilon, \rho I) \in H(I)$ implies $(\sigma, \varrho I) \in H(I)$, for all $(\varepsilon, \rho I), (\sigma, \varrho I) \in H(I)$.

Proposition 3.7. Every NI of NAT-A is a NAT-S.

Proof. Suppose that $(\chi(I), \curlyvee, (0,0))$ is neutrosophic AT-algebra. Let $w = (\varepsilon, \rho I) \in \chi(I)$ and $r = (\sigma, \varrho I) \in \chi(I)$ be arbitrary elements of $S(I)$, then $(w * r) = (\varepsilon, \rho I) \curlyvee (\sigma, \varrho I) = (\varepsilon \wedge \sigma, (\rho \wedge \varrho)I) \in S(I)$.

Definition 3.8. Let $(\chi(I), \curlyvee, (0,0))$ be neutrosophic AT-algebra and let $H(I)$ be a nonempty of $\chi(I)$. $H(I)$ is called **a neutrosophic AT-ideal of $\boldsymbol{\chi}$ (NAT-I)** if

1) $(0,0) \in H(I)$

2) $(\varepsilon, \rho I) \curlyvee ((\sigma, \varrho I) \curlyvee (\tau, \iota I)) \in H(I)$ and $(\sigma, \varrho I) \in H(I)$ implies $(\varepsilon, \rho I) \curlyvee (\tau, \iota I) \in H(I)$, for all $(\varepsilon, \rho I), (\sigma, \varrho I), (\tau, \iota I) \in H(I)$.

Proposition 3.9. Every NAT-I of NAT-A is an NI.

Proof. Suppose that $(\chi(I), \curlyvee, (0,0))$ is neutrosophic AT-algebra and $H(I)$is a neutrosophic

AT-ideal of $\chi(I)$. Let $w = (\varepsilon, \rho I)$, $r = (\sigma, \varrho I)$ and $t = (\tau, \iota I)$ be arbitrary elements of $\chi(I)$ such that

$\big(w * (r * t)\big) = (\varepsilon, \rho I)\ \Upsilon((\sigma, \varrho I)\Upsilon(\tau, \iota I)) \in H(I)$ and $(\sigma, \varrho I) \in H(I)$.

If $(\varepsilon, \rho I) = (0,0I)$, then $(0,0I)\ \Upsilon\big((\sigma, \varrho I)\Upsilon(\tau, \iota I)\big) = (\sigma, \varrho I)\ \Upsilon(\tau, \iota I) \in H(I)$

and $r = (\sigma, \varrho I) \in H(I)$, then $\big((0,0I)\Upsilon(\tau, \iota I)\big) = (\tau, \iota I) \in H(I)$.

Hence $H(I)$ is neutrosophic ideal.

Proposition 3.10. Every NAT-I of NAT-A is a NAT-S.

Proof. Suppose that $(\chi(I), \Upsilon, (0,0))$ is neutrosophic AT-algebra and $H(I)$is a neutrosophic AT-ideal of $\chi(I)$. Let $w = (\varepsilon, \rho I)$, $r = (\sigma, \varrho I)$ and $t = (\tau, \iota I)$ be arbitrary elements of $\chi(I)$ such that

$\big(w * (r * t)\big) = (\varepsilon, \rho I)\ \Upsilon((\sigma, \varrho I)\Upsilon(\tau, \iota I)) \in H(I)$ and $(\sigma, \varrho I) \in H(I)$.

If $(\sigma, \varrho I) = (0,0I)$, then $(\varepsilon, \rho I)\ \Upsilon\big((0,0I)\Upsilon(\tau, \iota I)\big) = (\varepsilon, \rho I)\ \Upsilon(\tau, \iota I) \in H(I)$.

Hence $H(I)$ is neutrosophic AT-subalgebra.

Definition 3.11. Let $(\chi; *, 0)$ be any AT-algebra, $(\chi(I), \Upsilon, (0,0))$ and $(\chi'(I), \Upsilon', (0', 0'))$ be two neutrosophic AT-algebras. A mapping $f: (\chi(I), \Upsilon, (0,0)) \to (\chi'(I), \Upsilon', (0', 0'))$ is called a **neutrosophic homomorphism** if $f((\nu, \varepsilon I)\Upsilon(\rho, \sigma I)) = f(\nu, \varepsilon I)\Upsilon' f(\rho, \sigma I)$, for all $(\nu, \varepsilon I), (\rho, \sigma I) \in \chi(I)$

Remark 3.12.

(1) If f is injective, then it is referred to as a neutrosophic monomorphism.

(2) If f is subjective, it is termed a neutrosophic epimorphism.

(3) If f is a bijection, it is referred to as a neutrosophic isomorphism.

A bijective neutrosophic homomorphism from $\chi(I)$ onto $\chi'(I)$ is termed a neutrosophic automorphism.

The subsequent theorem addresses the fundamental features of neutrosophic homomorphism.

Theorem 3.13. Letting $f: (\chi(I), \Upsilon, (0,0)) \to (\chi'(I), \Upsilon', (0', 0'))$ be a mapping of a neutrosophic AT-algebra $\chi(I)$ into an AT-algebra $\chi'(I)$, then:

(1) $f(0,0I) = (0', 0'I)$.

(2) If $(0,0I)$ serves as the identity in $\chi(I)$, then $f(0,0I)$ functions as the identity in $\chi'(I)$.

(3) $(\varepsilon, \rho I) \le (\sigma, \varrho I)$ implies $f(\varepsilon, \rho I) \le f(\sigma, \varrho I)$, for all $(\varepsilon, \rho I), (\sigma, \varrho I) \in \chi(I)$.

Proof.

Since $f: (\chi(I), \Upsilon, (0,0)) \to (\chi'(I), \Upsilon', (0', 0'))$ is a neutrosophic homomorphism.

(1) Since 0*0 = 0, then $f(0,0I) = f((0,0I)\Upsilon(0,0I))$= $f((0,0I))\ \Upsilon'\ f((0,0I))$= $(0', 0'I)$.

(2) Since $(0,0I)$ is the identity in $\chi(I)$ and $(0', 0'I)$ is the identity in $\chi'(I)$. From (AT_3),

$f(0,0I)\Upsilon(0',0'I) = f(0,0I) = (0',0'I)$ and

$f(0,0I)\Upsilon'(0',0'I) = f(0,0I)\Upsilon'(f(0,0I)\Upsilon' f(0,0I))$

$= f(0,0I)\Upsilon' f((0,0I)\Upsilon(0,0I))$

$= f(0,0I)\Upsilon' f(0,0I)$

$=(0',0'I)$

By (1), we get that $f(0,0I) = (0',0'I)$. This show that $f(0,0I)$ is the identity in $\chi'(I)$.

(3) Let $(\varepsilon,\rho I) \leq (\sigma,\varrho I)$. It follows that $(\sigma,\varrho I)\Upsilon(\varepsilon,\rho I) = (0,0I)$. So, (1) implies

$f(\sigma,\varrho I)\Upsilon' f(\varepsilon,\rho I) = f\big((\sigma,\varrho I)\Upsilon(\varepsilon,\rho I)\big) = f(0,0I) = (0',0'I)$.

Hence $f(\varepsilon,\rho I) \leq f(\sigma,\varrho I)$.⌂

Definition 3.14. Let $f\colon(\chi(I),\Upsilon,(0,0)) \to (\chi'(I),\Upsilon',(0',0'))$ be a mapping of a neutrosophic AT-algebra $\chi(I)$ into a neutrosophic AT-algebra $\chi'(I)$, and let $\beta(I) \subseteq \chi(I)$ and $\beta'(I) \subseteq \chi'(I)$.

1) The image of $\boldsymbol{\beta(I)}$ of $\boldsymbol{\chi(I)}$ under $\boldsymbol{f}$ is $f\big(\beta(I)\big) = \{f(\varepsilon,\rho I) | (\varepsilon,\rho I) \in \beta(I)\}$.

2) The inverse image of $\boldsymbol{\beta'(I)}$ of $\boldsymbol{\chi'(I)}$ is $f^{-1}\big(\beta'(I)\big) = \{(\varepsilon,\rho I) \in \chi(I) | f(\varepsilon,\rho I) \in \beta'(I)\}$.

3) The kernel of $\boldsymbol{f}$ is $\{(\varepsilon,\rho I) \in \chi(I) | f(\varepsilon,\rho I) = (0',0'I)\}$ denoted by ker f .

Theorem 3.15. Letting $f\colon(\chi(I),\Upsilon,(0,0)) \to (\chi'(I),\Upsilon',(0',0'))$ be a mapping of a neutrosophic AT-algebra $\chi(I)$ into a neutrosophic AT-algebra $\chi'(I)$, then

(1) If $\beta(I)$ is a neutrosophic AT-subalgebra of $\chi(I)$, then $f(\beta(I))$ is a neutrosophic AT-subalgebra of $\chi'(I)$.

(2) If $\beta'(I)$ constitutes a neutrosophic AT-subalgebra of $\chi'(I)$, then $f^{-1}(\beta'(I))$ qualifies as a neutrosophic AT-subalgebra of $\chi(I)$.

(3) Im (f) is a neutrosophic AT-subalgebra of $\chi'(I)$.

Proof.

Since $f\colon(\chi(I),\Upsilon,(0,0)) \to (\chi'(I),\Upsilon',(0',0'))$ is a neutrosophic homomorphism.

(1) Let $\beta(I)$ be a neutrosophic AT-subalgebra of $\chi(I)$ and $(\varepsilon,\rho I),(\sigma,\varrho I) \in f(\beta(I))$, then there exist $(\tau,\iota I)$, $(\nu,\epsilon I) \in \beta(I)$ such that $(\varepsilon,\rho I) = f(\tau,\iota I)$ and $(\sigma,\varrho I) = f(\nu,\epsilon I)$. Since $(\varepsilon,\rho I)\Upsilon'(\sigma,\varrho I)$=$f(\tau,\iota I)\Upsilon' f(\nu,\epsilon I)$

$= f((\tau,\iota I)\Upsilon(\nu,\epsilon I)) \in f(\beta(I))$. Thus $f(\beta(I))$ is a neutrosophic AT-subalgebra of $\chi'(I)$. $\beta(I)$ contains a proper subset which is a neutrosophic AT-algebra.

(2) Let $\beta'(I)$ be a neutrosophic AT-subalgebra of $\chi'(I)\Upsilon$ and $(\varepsilon,\rho I),(\sigma,\varrho I) \in f^{-1}\big(\beta'(I)\big)$, then $f(\varepsilon,\rho I) = (\tau,\iota I)$ and $f(\sigma,\varrho I) = (\nu,\epsilon I)$for some $(\tau,\iota I),(\nu,\epsilon I) \in \beta'(I)$. Thus $f((\varepsilon,\rho I)\Upsilon(\sigma,\varrho I)$ =$f(\varepsilon,\rho I)\Upsilon' f(\sigma,\varrho I)$=$(\tau,\iota I)\Upsilon'(\nu,\epsilon I) \in \beta'(I)$, as $\beta'(I)$ is a neutrosophic AT-subalgebra. Hence$(\varepsilon,\rho I)\Upsilon(\sigma,\varrho I) \in f^{-1}(\beta'(I))$.

$\beta'(I)$ contains a proper subset which is a neutrosophic AT-algebra.

(3) Let $(\varepsilon, \rho I)$, $(\sigma, \varrho I)$ $\in$Im(f), then there exist $(\tau, \iota I), (\nu, \varepsilon I) \in \chi(I)$ such that $(\varepsilon, \rho I) = f((\tau, \iota I))$ and $(\sigma, \varrho I) = f((\nu, \varepsilon I))$, so

$(\varepsilon, \rho I) \Upsilon' (\sigma, \varrho I)= f(\tau, \iota I) \Upsilon' f(\nu, \varepsilon I) = f((\tau, \iota I) \Upsilon (\nu, \varepsilon I))\in$Im$(f)$.This proves that Im$(f)$ is a neutrosophic AT-subalgebra of $\chi'(I)$.

Im(f) contains a proper subset which is a neutrosophic AT-algebra.

In general, Im(f) may not be a neutrosophic AT-ideal .

Theorem 3.16. Let $f: (\chi(I), \Upsilon, (0,0)) \rightarrow (\chi'(I), \Upsilon', (0', 0'))$ be a mapping of a neutrosophic AT-algebra $\chi(I)$ into a neutrosophic AT-algebra $\chi'(I)$, then

(1) If $\beta(I)$ is a neutrosophic ideal of $\chi(I)$, then $f(\beta(I))$ is a neutrosophic ideal of $\chi'(I)$.

(2) If $\beta'(I)$ is a neutrosophic ideal of $\chi'(I)$, then $f^{-1}(\beta'(I))$ is a neutrosophic ideal of $\chi(I)$.

Proof.

Since $f: (\chi(I), \Upsilon, (0,0)) \rightarrow (\chi'(I), \Upsilon', (0', 0'))$ is a neutrosophic homomorphism.

(1) Let $\beta(I)$ be a neutrosophic AT-ideal of $\chi(I)$. We see that $(0,0I) \in \beta(I)$, and by theorem (3.13(1)),$(0', 0'I) = f(0,0I) \in f(\beta(I))$, so $(0', 0'I) \in f(\beta(I))$.

New, let $\beta(I)$ be a neutrosophic ideal of $\chi(I)$ and $(\varepsilon, \rho I)\Upsilon'(\sigma, \varrho I), (\varepsilon, \rho I) \in f(\beta(I))$, then there exist $(\tau, \iota I), (\nu, \varepsilon I) \in \beta(I)$ such that $(\varepsilon, \rho I) = f(\tau, \iota I)$ and $(\sigma, \varrho I) = f(\nu, \varepsilon I)$.

Since $(\varepsilon, \rho I)\Upsilon'^{(\sigma, \varrho I)}$=$f(\tau, \iota I)\Upsilon' f(\nu, \varepsilon I) = f((\tau, \iota I)\Upsilon(\nu, \varepsilon I))$. Thus $(\tau, \iota I)\Upsilon(\nu, \varepsilon I) \in \beta(I)$ and $(\tau, \iota I) \in \beta(I)$.

But $\beta(I)$ is a neutrosophic ideal, then $(\nu, \varepsilon I) \in \beta(I)$ implies that $(\sigma, \varrho I) \in f(\beta(I))$. Thus $f(\beta(I))$ is a neutrosophic ideal of $\chi'(I)$.

(2) Let $\beta'(I)$ be a neutrosophic AT-ideal in $\chi'(I)$. Then $(0', 0'I) \in \beta'(I)$, we get that $(0,0I) = f^{-1}(0', 0'I) \in f^{-1}(\beta'(I))$.

Now, let $\beta'(I)$ be a neutrosophic ideal of $\chi'(I)$ and $(\varepsilon, \rho I)\Upsilon(\sigma, \varrho I), (\varepsilon, \rho I) \in f^{-1}(\beta'(I))$, then $f(\varepsilon, \rho I) = (\tau, \iota I)$ and $f(\sigma, \varrho I) = (\nu, \varepsilon I)$ for some $(\tau, \iota I), (\nu, \varepsilon I) \in \beta'(I)$. Thus $f((\varepsilon, \rho I)\Upsilon(\sigma, \varrho I)$ =$f(\varepsilon, \rho I)\Upsilon' f(\sigma, \varrho I)$=$(\tau, \iota I)\Upsilon'(\nu, \varepsilon I)$. Thus $(\tau, \iota I)\Upsilon(\nu, \varepsilon I) \in \beta'(I)$ and $(\tau, \iota I) \in \beta'(I)$.

But $\beta'(I)$ is a neutrosophic ideal, then $(\nu, \varepsilon I) = f(\sigma, \varrho I) \in \beta'(I)$ implies that $(\sigma, \varrho I) \in f^{-1}(\beta'(I))$. Thus $f^{-1}(\beta'(I))$ is a neutrosophic ideal of $\chi(I)$.

Theorem 3.17. Let $f: (\chi(I), \Upsilon, (0,0)) \rightarrow (\chi'(I), \Upsilon', (0', 0'))$ be a mapping of a neutrosophic AT-algebra $\chi(I)$ into a neutrosophic AT-algebra $\chi'(I)$, then

(1) If $\beta(I)$ is a neutrosophic AT-ideal of $\chi(I)$, then $f(\beta(I))$ is a neutrosophic AT-ideal of $\chi'(I)$.

(2) If $\beta'(I)$ is a neutrosophic AT-ideal of $\chi'(I)$, then $f^{-1}(\beta'(I))$ is a neutrosophic AT-ideal of $\chi(I)$.

(3) ker f is a neutrosophic AT-ideal of $\chi(I)$.

Proof.

Since $f: (\chi(I), \Upsilon, (0,0)) \to (\chi'(I), \Upsilon', (0', 0'))$ is a neutrosophic homomorphism.

(1) Let $\beta(I)$ be a neutrosophic AT-ideal of $\chi(I)$. We see that $(0,0I) \in \beta(I)$, and by theorem (3.13(1)),$(0', 0'I) = f(0,0I) \in f(\beta(I))$, so $(0', 0'I) \in f(\beta(I))$.

Now, assume that Let $(\varepsilon, \rho I)$, $(\sigma, \varrho I)$, $(\tau, \iota I) \in \beta(I)$, then $f((\varepsilon, \rho I))\ \Upsilon'\ (f((\sigma, \varrho I))\ \Upsilon'\ f((\tau, \iota I))\ ) \in f(\beta(I))$ and $f\ ((\sigma, \varrho I)) \in f(\beta(I))$, but $(\varepsilon, \rho I)\ \Upsilon((\sigma, \varrho I)\Upsilon(\tau, \iota I))$, $(\varepsilon, \rho I) \in \beta(I)$. Since $\beta(I)$ is a neutrosophic AT-ideal of $\chi(I)$, it follows that $(\varepsilon, \rho I)\Upsilon(\tau, \iota I) \in \beta(I)$. Thus $f\ ((\varepsilon, \rho I)\Upsilon(\tau, \iota I)) = f(\varepsilon, \rho I)\ \Upsilon' f\ (\tau, \iota I) \in f\ (\beta(I))$. Hence $f\ (\beta(I))$ is a neutrosophic AT-ideal of $\chi'(I)$.

(2) Let $\beta'(I)$ be a neutrosophic AT-ideal in $\chi'(I)$. Then $(0', 0'I) \in \beta'(I)$, we get that $(0,0I) = f^{-1}(0', 0'I) \in f^{-1}(\beta'(I))$.

For any $(\varepsilon, \rho I)$, $(\sigma, \varrho I)$, $(\tau, \iota I) \in \chi(I)$, let $(\varepsilon, \rho I)\Upsilon\big((\sigma, \varrho I)\Upsilon(\tau, \iota I)\big) \in f^{-1}(\beta'^{(I)})$ and $(\sigma, \varrho I) \in f^{-1}\ (\beta'^{(I)})$. It follows that $f\ ((\varepsilon, \rho I)\Upsilon'(\ f\ (\sigma, \varrho I)\Upsilon'\ f$ $(\tau, \iota I)\ )$=$f((\varepsilon, \rho I)\Upsilon((\sigma, \varrho I)\Upsilon(\tau, \iota I)) \in \beta'(I)$ and $f\ (\sigma, \varrho I) \in \beta'(I)$. Since $\beta'(I)$ is a neutrosophic AT-ideal of $\chi'(I)$, we obtain that $f\ ((\varepsilon, \rho I)\Upsilon(\tau, \iota I))$ =$f\ ((\varepsilon, \rho I))\ \Upsilon' f(\tau, \iota I)) \in \beta'(I)$. Consequently $(\sigma, \varrho I) \in f^{-1}(\beta(I))$, proving that$f^{-1}\ (\beta'(I))$ is a neutrosophic AT-ideal of $\chi(I)$.

(3) It is clear that $\ker f \subseteq \chi(I)$. Since $f\ (0,0I) = (0', 0'I)$, so $(0,0I) \in \ker f$. It follows that $\ker f \neq \emptyset$. Let $(\varepsilon, \rho I)\Upsilon((\sigma, \varrho I)\Upsilon(\tau, \iota I)) \in \ker f$ and $(\sigma, \varrho I) \in \ker f$, then $f((\varepsilon, \rho I)\Upsilon\big((\sigma, \varrho I)\Upsilon(\tau, \iota I)\big) = (0', 0'I)$, $f(\sigma, \varrho I) = (0', 0'I)$. Since

$(0', 0'I) = f\left((\varepsilon, \rho I)\Upsilon\big((\sigma, \varrho I)\Upsilon(\tau, \iota I)\big)\right) = f(\varepsilon, \rho I)\Upsilon'\ f\big((\sigma, \varrho I)\Upsilon(\tau, \iota I)\big)$

$= f(\varepsilon, \rho I)\Upsilon'(\ f(\sigma, \varrho I)\Upsilon' f(\tau, \iota I))$, but $f(\sigma, \varrho I) = (0', 0'I)$, then

$(f(\varepsilon, \rho I)\ \Upsilon'\ f(\tau, \iota I)) = (0', 0'I)$ imply $f((\varepsilon, \rho I)\Upsilon(\tau, \iota I))$=$f(\varepsilon, \rho I)\ \Upsilon'\ f(\tau, \iota I) = (0', 0'I)$.

$(\varepsilon, \rho I)\Upsilon(\tau, \iota I) \in \ker f$. Consequently, $\ker f$ constitutes a neutrosophic AT-ideal of $\chi(I)$.

4. APPENDIX-ALGORITHMS

This appendix includes the requisite algorithms.

Algorithm for AT-algebras

Input (χ : set, Y : binary operation)

Determine whether χ constitutes an AT-algebra.

If χ = φ, then proceed to (1.);

End If

If 0 is not an element of χ, proceed to (1.);

End If

Stop: = false;

i :□1;

While i □ |χ| and not (Stop) do

If 0 Y ε_i□ ε_i then

Stop: = true;

End

If j :□1

While j □ |χ| and not (Stop) do

If ε_i Y 0 □ 0 then

Stop: = true;

End If

End If

k :□1 While k □ |χ| and not (Stop) do

If (ε_i Y ρ_j) Y ((ρ_j Y σ_k) Y (ε_i Y σ_k)) □ 0 then

Stop: = true;

End If

End If While

End If While

End If While

If Stop then

(1.) Output (“ χ is not an AT-algebra”)

Else

Output (“ χ is an AT-algebra”)

End If

End

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