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# Some Application for a Fuzzy Differential Equation and Solve by Runge-Kutta Method

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***Abstract.*** *In this article, the starting condition was defined using a fuzzy initial value problem (IVP). Additionally, we discussed various methods for solving fuzzy differential equations, including the modified two-step Simpson method and Runge-Kutta of orders (two, three, four, five, and six). For each method, we provided a numerical example and the known convergence rates of the solutions. Then we discussed the comparison of the solutions of all methods, using computer software to offer rough solutions for the Runge Kutta method. And take some application solve by Runge-Kutta in physics and medical*

***Keywords:*** *Fuzzy differential equation, modified two-step Simpson method, Runge-Kutta method, initial value problem.*

## 1. INTRODUCTION

Zadeh was the first to introduce the idea of a fuzzy set in [1]. Since then, the theory has developed to the point where it is now acknowledged as a unique discipline of applied mathematics. Because it offers a natural method for expressing dynamical systems under uncertainty, the theory of differential equations with fuzzy coefficients is significant in modeling scientific and engineering challenges. Fuzzy differential equations (FDE) and fuzzy initial value problems (IVP) are studied in [2], [3], and [4]. Theoretical and numerical solutions to FDEs were being discussed by a number of academics [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16]. We presented the Runge Kutta technique of order (2, 3, 4, 5 and 6) with a modified 2-step Simpson method to solve some examples of FDE, and ranking from best to least of a solution (sol) of all methods. Throughout this study, we proposed the Runge Kutta technique to solve FDE and employing computer software to supply approximate sols of specific instances.

## 2. METHODOLOGY STUDY

### Fuzzy initial value problem

Take the following first order Fuzzy IVP of DE:

$$\begin{cases} Y'(ŧ) = f(ŧ, Y(ŧ)), ŧ \in [ŧ_0, T] \\ Y(ŧ_0) = Y_0 \end{cases}$$

When Y be fuzzy function of $ŧ, f(ŧ, Y)$ with variable ŧ and a fuzzy var. Y & $Y'$ are fuzzy derivative of Y and $Y(ŧ_0) = Y_0$ be the triangular fuzzy number [17].

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A fuzzy function y is written as follows: $Y = [\underline{Y}, \bar{Y}]$. So a r-level set on $Y(t)$ when $t \in [t_0, T]$ was $[Y(t)]_r = [\underline{Y}(t;r), \bar{Y}(t;r)], [Y(t_0)]_r = \left[\underline{Y}(t_0;r), \bar{Y}(t_0;r)\right], r \in (0,1]$, it is mean $f(t,Y) = [\underline{f}(t,Y), \bar{f}(t,Y)]$ and $\underline{f}(t,Y) = F[t, \underline{Y}, \bar{Y}]$, $\bar{f}(t,Y) = G[t, \underline{Y}, \bar{Y}]$, Since $Y' = f(t,Y)$ we get

$$\underline{f}(t, Y(t); r) = F[t, \underline{Y}(t;r), \bar{Y}(t;r)]$$
$$\bar{f}(t, Y(t); r) = G[t, \underline{Y}(t;r), \bar{Y}(t;r)]$$

And then $f(t, Y(t))(s) = \sup\{Y(t)(\tau) \setminus s = f(t,\tau)\}, s \in R$

So the fuzzy number $f(t, Y(t))$ follows that

$$[f(t, Y(t))]_r = [\underline{f}(t, Y(t); r), \bar{f}(t, Y(t); r)], r \in (0,1]$$
$$\text{where } \underline{f}(t, Y(t); r) = \min\{f(t,u) \mid u[Y(t)]_r\}$$
$$\bar{f}(t, Y(t); r) = \max\{f(t,u) \mid u[Y(t)]_r\}$$

The function $f: R \to R_F$ is known as a F. continuous fun. If for each constant $t_0 \in R$ with $\varepsilon > 0, \delta > 0$ so $|t - t_0| < \delta$ implies $D[f(t), f(t_0)] < \varepsilon$ exists [18].

In D, the fuzzy function toke is continuous, as well as the continuity of such $f(t, Y(t); r)$ assures the definition's existence of $f(t, Y(t); r)$ for $t \in [t_0, T]$ with $r \in [0,1]$. So, the functions G and F can also be definitive.

**A modified two-step Simpson method**

Assuming “$Y = [\underline{Y}, \bar{Y}]$” is an exact sol with “$Y = [\underline{Y}, \bar{Y}]$” is the 2-step modified Simpson method's approximate sol initial value Equation [19]. Assume,

$$"[Y(t)]_r = [\underline{Y}(t;r), \bar{Y}(t;r)]\,,\ [Y(t)]_r = [\underline{Y}(t;r), \bar{Y}(t;r)]"$$

It's also worth noting that a value $r$ be constant during each integration phase. At $t_n$, the precise and approximation sols are indicated by

$$"[Y_n]_r = \left[\underline{Y}_n(r), \bar{Y}_n(r)\right],\ [Y_n]_r = \left[\underline{Y}_n(r), \bar{Y}_n(r)\right]\ (0 \le n \le N)"$$

The grid locations are determined, accordingly.

$$h = \frac{T - t_0}{N},\ t_i = t_0 + ih\ ,"0 \le i \le N"$$

We get the following results using the modified Simpson method:

$$\underline{Y}_{n+1}(r) = \underline{Y}_{n-1}(r) + \frac{h}{3}F\left[t_{n-1}, \underline{Y}_{n-1}(r), \bar{Y}_{n-1}(r)\right] + \frac{4h}{3}F\left[t_n, \underline{Y}_n(r), \bar{Y}_n(r)\right]$$
$$+\frac{h}{3}F\left[t_{n+1}, \underline{Y}_n(r) + hF\left[t_n, \underline{Y}_n(r), \bar{Y}_n(r)\right], \bar{Y}_n(r) + hG\left[t_n, \underline{Y}_n(r), \bar{Y}_n(r)\right]\right]$$
$$+h^3\underline{A}(r)$$

And

$$\bar{Y}_{n+1}(r) = \bar{Y}_{n-1}(r) + \frac{h}{3}G[t_{n-1}, \underline{Y}_{n-1}(r), \bar{Y}_{n-1}(r)] + \frac{4h}{3}G[t_n, \underline{Y}_n(r), \bar{Y}_n(r)]$$
$$+\frac{h}{3}G\left[t_{n+1}, \underline{Y}_n(r) + hF[t_n, \underline{Y}_n(r), \bar{Y}_n(r)], \bar{Y}_n(r) + hG[t_n, \underline{Y}_n(r), \bar{Y}_n(r)]\right]$$
$$+h^3\bar{A}(r)$$

When $A = [\underline{A}, \bar{A}], [A]_r = [\underline{A}(r), \bar{A}(r)]$ and

$$[A]_r = \left[\frac{1}{6}f'(\xi_2, Y(\xi_2)) \cdot f_Y(t_{i+1}, \xi_3) - \frac{h^2}{90}f^{(4)}(\xi_1, Y(\xi_1))\right]_r.$$

$$\Rightarrow \quad \underline{Y}_{n+1}(r) = \underline{Y}_{n-1}(r) + \frac{h}{3}F[t_{n-1}, \underline{Y}_{n-1}(r), \bar{Y}_{n-1}(r)] + \frac{4h}{3}F[t_n, \underline{Y}_n(r), \bar{Y}_n(r)]$$
$$+\frac{h}{3}F\left[t_{n+1}, \underline{Y}_n(r) + hF[t_n, \underline{Y}_n(r), \bar{Y}_n(r)], \bar{Y}_n(r) + hG[t_n, \underline{Y}_n(r), \bar{Y}_n(r)]\right]$$

And

$$\bar{Y}_{n+1}(r) = \bar{Y}_{n-1}(r) + \frac{h}{3}G[t_{n-1}, \underline{Y}_{n-1}(r), \bar{Y}_{n-1}(r)] + \frac{4h}{3}G[t_n, \underline{Y}_n(r), \bar{Y}_n(r)]$$
$$+\frac{h}{3}G\left[t_{n+1}, \underline{Y}_n(r) + hF[t_n, \underline{Y}_n(r), \bar{Y}_n(r)], \bar{Y}_n(r) + hG[t_n, \underline{Y}_n(r), \bar{Y}_n(r)]\right]$$

**Runge-Kutta of order two**

Let an exact sol $[Y(t)]_r = [\underline{Y}(t;r), \bar{Y}(t;r)]$, be approximate by $[Y(t)]_r = [\underline{Y}(t;r), \bar{Y}(t;r)]$ [20]. A places on the grid where the sols are computed are $h = \frac{T-t_0}{N}, t_i = t_0 + ih; 0 \le i \le N$, then we define:

$$\underline{Y}(t_{n+1}, r) - \underline{Y}(t_n, r) = h\left[\frac{\underline{k_1^2}(t_n, Y(t_n, r)) + \underline{k_2^2}(t_n, Y(t_n, r))}{\underline{k_1}(t_n, Y(t_n, r)) + \underline{k_2}(t_n, Y(t_n, r))}\right]$$

When

$$k_1 = hF[t_n, \underline{Y}(t_n, r), \bar{Y}(t_n, r)]$$
$$k_2 = hF\left[t_n + h, \underline{Y}(t_n, r) + \underline{k_1}(t_n, Y(t_n, r)), \bar{Y}(t_n, r) + \overline{k_1}(t_n, Y(t_n, r))\right]$$

And $\bar{Y}(t_{n+1}, r) - \bar{Y}(t_n, r) = h\left[\frac{\overline{k_1^2}(t_n, Y(t_n, r)) + \overline{k_2^2}(t_n, Y(t_n, r))}{\overline{k_1}(t_n, Y(t_n, r)) + \overline{k_2}(t_n, Y(t_n, r))}\right]$

When
$$k_1 = hG[t_n, \underline{Y}(t_n, r), \bar{Y}(t_n, r)]$$
$$k_2 = hG\left[t_n + h, \underline{Y}(t_n, r) + \underline{k_1}(t_n, Y(t_n, r)), \bar{Y}(t_n, r) + \overline{k_1}(t_n, Y(t_n, r))\right]$$

So can be define $G(t_n, Y(t_n, r)) = h\left[\frac{\overline{k_1^2}(t_n, Y(t_n, r)) + \overline{k_2^2}(t_n, Y(t_n, r))}{\overline{k_1}(t_n, Y(t_n, r)) + \overline{k_2}(t_n, Y(t_n, r))}\right]$

So we get:

$$\underline{Y}(t_{n+1}, r) = \underline{Y}(t_n, r) + F[t_n, Y(t_n, r)]]$$
$$\bar{Y}(t_{n+1}, r) = \bar{Y}(t_n, r) + G[t_n, Y(t_n, r)]$$

and

$$\underline{Y}(t_{n+1}, r) = \underline{Y}(t_n, r) + F[t_n, Y(t_n, r)]$$
$$\bar{Y}(t_{n+1}, r) = \bar{Y}(t_n, r) + G[t_n, Y(t_n, r)]$$

clearly $Y(t; r)$ and $\bar{Y}(t; r)$ converge to $\underline{Y}(t; r)$ and $\bar{Y}(t; r)$ whenever $h \to 0$

**Runge-Kutta of order three**

Let that we have a fuzzy IVP $Y'(t) = f(t, Y(t))Y(t_0) = Y_0$ [21]. All Runge-Kutta techniques are based on expressing the difference between the value of y at $t_{n+1}$ and $t_n$ as $Y_{n+1} - Y_n = \sum_{i=0}^{m} w_i k_i$

When $w_i$ 's are constant for all i and $k_i = hf\left(t_n + a_i h, Y_n + \sum_{j=1}^{i-1} c_{ij} k_j\right)$

Assume $Y(t_{n+1}) = Y(t_n) + \frac{h}{2}\left[\frac{k_1^2 + k_2^2}{k_1 + k_2} + \frac{k_2^2 + k_3^2}{k_2 + k_3}\right]$, When

$$k_1 = hf\left(t_n, Y(t_n)\right)$$
$$k_2 = hf(t_n + a_1, Y(t_n) + a_1 k_1)$$
$$k_3 = hf(t_n + a_2, Y(t_n) + a_2 k_2)$$

With the parameters $a_1, a_2$ were selected to produce $Y_{n+1}$ closer to $Y(t_{n+1})$. The parameter values $a_1 = \frac{2}{3}, a_2 = \frac{2}{3}$

Assume that an exact sol $[Y(t)]_r = [\underline{Y}(t; r), \bar{Y}(t; r)]$, be approximated by $[Y(t)]_r = [\underline{Y}(t; r), \bar{Y}(t; r)]$. A places on the grid where the sols are computed are $h = \frac{T - t_0}{N}$, $t_i = t_0 + ih; 0 \le i \le N$. Now can be define:

$$\underline{Y}(t_{n+1}, r) - \underline{Y}(t_n, r) = \frac{h}{2}\left[\frac{k_1^2\left(t_n, Y(t_n, r)\right) + \underline{k_2}^{\,2}\left(t_n, Y(t_n, r)\right)}{\underline{k_1}\left(t_n, Y(t_n, r)\right) + \underline{k_2}\left(t_n, Y(t_n, r)\right)} + \frac{\underline{k}_2^2\left(t_n, Y(t_n, r)\right) + \underline{k_3}^{\,2}\left(t_n, Y(t_n, r)\right)}{\underline{k_2}\left(t_n, Y(t_n, r)\right) + \underline{k_3}\left(t_n, Y(t_n, r)\right)}\right]$$

When

$$k_1 = hF\left[t_n, \underline{Y}(t_n, r), \bar{Y}(t_n, r)\right]$$
$$k_2 = hF\left[t_n + \frac{2}{3}, \underline{Y}(t_n, r) + \frac{2}{3}k_1\left(t_n, Y(t_n, r)\right), \bar{Y}(t_n, r) + \frac{2}{3}\overline{k_1}\left(t_n, Y(t_n, r)\right)\right]$$
$$k_3 = hF\left[t_n + \frac{2}{3}, \underline{Y}(t_n, r) + \frac{2}{3}k_2\left(t_n, Y(t_n, r)\right), \bar{Y}(t_n, r) + \frac{2}{3}\overline{k_2}\left(t_n, Y(t_n, r)\right)\right]$$

With

$$\bar{Y}(t_{n+1},r) - \bar{Y}(t_n,r) = \frac{h}{2}\left[\frac{\overline{k_1^2}(t_n,Y(t_n,r)) + \overline{k_2^2}(t_n,Y(t_n,r))}{\overline{k_1}(t_n,Y(t_n,r)) + \overline{k_2}(t_n,Y(t_n,r))} + \frac{\overline{k_2^2}(t_n,Y(t_n,r)) + \overline{k_3^2}(t_n,Y(t_n,r))}{\overline{k_2}(t_n,Y(t_n,r)) + \overline{k_3}(t_n,Y(t_n,r))}\right]$$

When

$$k_1 = hG[t_n, \underline{Y}(t_n,r), \bar{Y}(t_n,r)]$$
$$k_2 = hG\left[t_n + \frac{2}{3}, \underline{Y}(t_n,r) + \frac{2}{3}k_1(t_n,Y(t_n,r)), \bar{Y}(t_n,r) + \frac{2}{3}\overline{k_1}(t_n,Y(t_n,r))\right]$$
$$k_3 = hG\left[t_n + \frac{2}{3}, \underline{Y}(t_n,r) + \frac{2}{3}k_2(t_n,Y(t_n,r)), \bar{Y}(t_n,r) + \frac{2}{3}\overline{k_2}(t_n,Y(t_n,r))\right]$$

We also define:

$$F(t_n,Y(t_n,r)) = \frac{h}{2}\left[\frac{\underline{k_1^2}(t_n,Y(t_n,r)) + \underline{k_2^2}(t_n,Y(t_n,r))}{\underline{k_1}(t_n,Y(t_n,r)) + \underline{k_2}(t_n,Y(t_n,r))} + \frac{\underline{k_2^2}(t_n,Y(t_n,r)) + \underline{k_3}^2(t_n,Y(t_n,r))}{\underline{k_2}(t_n,Y(t_n,r)) + \underline{k_3}(t_n,Y(t_n,r))}\right]$$

$$G(t_n,Y(t_n,r)) = \frac{h}{2}\left[\frac{\overline{k_1^2}(t_n,Y(t_n,r)) + \overline{k_2^2}(t_n,Y(t_n,r))}{\overline{k_1}(t_n,Y(t_n,r)) + \overline{k_2}(t_n,Y(t_n,r))} + \frac{\overline{k_2^2}(t_n,Y(t_n,r)) + \overline{k_3^2}(t_n,Y(t_n,r))}{\overline{k_2}(t_n,Y(t_n,r)) + \overline{k_3}(t_n,Y(t_n,r))}\right]$$

So we get
$$\underline{Y}(t_{n+1},r) = \underline{Y}(t_n,r) + F[t_n,Y(t_n,r)]$$
$$\bar{Y}(t_{n+1},r) = \bar{Y}(t_n,r) + G[t_n,Y(t_n,r)]$$

With
$$\underline{Y}(t_{n+1},r) = \underline{Y}(t_n,r) + F[t_n,Y(t_n,r)]$$
$$\bar{Y}(t_{n+1},r) = \bar{Y}(t_n,r) + G[t_n,Y(t_n,r)]$$

It's obvious $Y(t;r)$ , $\bar{Y}(t;r)$ converge to $\underline{Y}(t;r)$ , $\bar{Y}(t;r)$ , respectively where $h \to 0$

**Runge-Kutta of order four**

The first-order FDE is written in the following form: $\begin{matrix}\dot{Y}(t) = f(t,Y) \\ Y(t_o) = Y_o\end{matrix}$ [22]. An exact sol would be: $[Y(t_n)]_r = [\underline{Y}(t_n;r), \bar{Y}(t_n;r)]$ an approximate sol is as follows: $[Y(t_n)]_r = [Y(t_n;r), \bar{Y}(t_n;r)]$.

The Runge-Kutta technique of order four was used.

$$[Y(t_n)]_r = [\underline{Y}(t_n;r), \bar{Y}(t_n;r)$$
$$\underline{Y}(t_{n+1};r) = \underline{Y}(t_n;r) + \sum_{j=1}^{4} w_j k_{j,1}(t_n,Y(t_n,r))$$
$$\bar{Y}(t_{n+1};r) = \bar{Y}(t_n;r) + \sum_{j=1}^{4} w_j k_{j,2}(t_n,Y(t_n,r))$$

When $k_{j,1}, k_{j,2}$ describe the following:

$$k_{1,1}(t_n,Y(t_n;r)) = \min h\left\{Y(t_n,u) \mid u \in \left(\underline{Y}(t_n;r), \bar{Y}(t_n;r)\right)\right\}$$

$$k_{1,2}\big(t_n, Y(t_n; r)\big) = \max h\left\{Y(t_n, u): u \in \big(\underline{Y}(t_n; r), \bar{Y}(t_n; r)\big)\right\}$$

$$k_{2,1}\big(t_n, Y(t_n; r)\big) = \min h\left\{Y\left(t_n + \frac{h}{2}, u\right): u \in \Big(q_{1,1}\big(t_n; Y(t_n, r)\big), q_{1,2}\big(t_n; Y(t_n, r)\big)\Big)\right\}$$

$$k_{2,2}\big(t_n, Y(t_n; r)\big) = \max h\left\{Y\left(t_n + \frac{h}{2}, u\right): u \in \Big(q_{1,1}\big(t_n; Y(t_n, r)\big), q_{1,2}\big(t_n; Y(t_n, r)\big)\Big)\right\}$$

$$k_{3,1}\big(t_n, Y(t_n; r)\big) = \min h\left\{Y\left(t_n + \frac{h}{2}, u\right): u \in \Big(q_{2,1}\big(t_n; Y(t_n, r)\big), q_{2,2}\big(t_n; Y(t_n, r)\big)\Big)\right\}$$

$$k_{3,2}\big(t_n, Y(t_n; r)\big) = \max h\left\{Y\left(t_n + \frac{h}{2}, u\right): u \in \Big(q_{2,1}\big(t_n; Y(t_n, r)\big), q_{2,2}\big(t_n; Y(t_n, r)\big)\Big)\right\}$$

$$k_{4,1}\big(t_n, Y(t_n; r)\big) = \min h\left\{Y\left(t_n + \frac{h}{2}, u\right): u \in \Big(q_{3,1}\big(t_n; Y(t_n, r)\big), q_{3,2}\big(t_n; Y(t_n, r)\big)\Big)\right\}$$

$$k_{4,2}\big(t_n, Y(t_n; r)\big) = \max h\left\{Y\left(t_n + \frac{h}{2}, u\right): u \in \Big(q_{3,1}\big(t_n; Y(t_n, r)\big), q_{3,2}\big(t_n; Y(t_n, r)\big)\Big)\right\}$$

When:

$$\begin{aligned}
q_{1,1}\big(t_n; Y(t_n, r)\big) &= \underline{Y}(t_n, r) + \frac{h}{2} k_{1,1}\big(t_n, Y(t_n; r)\big)\\
q_{1,2}\big(t_n; Y(t_n, r)\big) &= \bar{Y}(t_n, r) + \frac{h}{2} k_{1,2}\big(t_n, Y(t_n; r)\big)\\
q_{2,1}\big(t_n; Y(t_n, r)\big) &= \underline{Y}(t_n, r) + \frac{h}{2} k_{2,1}\big(t_n, Y(t_n; r)\big)\\
q_{2,2}\big(t_n; Y(t_n, r)\big) &= \bar{Y}(t_n, r) + \frac{h}{2} k_{2,2}\big(t_n, Y(t_n; r)\big)\\
q_{3,1}\big(t_n; Y(t_n, r)\big) &= \underline{Y}(t_n, r) + \frac{h}{2} k_{3,1}\big(t_n, Y(t_n; r)\big)\\
q_{3,2}\big(t_n; Y(t_n, r)\big) &= \bar{Y}(t_n, r) + \frac{h}{2} k_{3,2}\big(t_n, Y(t_n; r)\big)
\end{aligned}$$

Using just the initial condition, we can now calculate:

$$\begin{aligned}
\underline{Y}(t_{n+1}; r) = \underline{Y}(t_n; r)&\\
&+ \frac{1}{6}\Big(k_{1,1}\big(t_n, Y(t_n; r)\big) + 2k_{2,1}\big(t_n, Y(t_n; r)\big) + 2k_{3,1}\big(t_n, Y(t_n; r)\big)\\
&+ k_{4,2}\big(t_n, Y(t_n; r)\big)
\end{aligned}$$

$$\begin{aligned}
\bar{Y}(t_{n+1}; r) = \bar{Y}(t_n; r)&\\
&+ \frac{1}{6}\Big(k_{1,2}\big(t_n, Y(t_n; r)\big) + 2k_{2,2}\big(t_n, Y(t_n; r)\big) + 2k_{3,2}\big(t_n, Y(t_n; r)\big)\\
&+ k_{4,2}\big(t_n, Y(t_n; r)\big)
\end{aligned}$$

A sol at $t_n$ where: $0 \le n \le N$, $a = t_o \le t_1 \le t_2 \le \cdots \le t_n = b$, and $h = \frac{b-a}{N} = t_{n+1} - t_n$,

$$\underline{Y}(t_{n+1}; r) = \underline{Y}(t_n; r) + \frac{1}{6} F[t_n, Y(t_n; r)]$$

$$\bar{Y}(t_{n+1}; r) = \bar{Y}(t_n; r) + \frac{1}{6} G[t_n, Y(t_n; r)],$$

$$\underline{Y}(t_{n+1}; r) = \underline{Y}(t_n; r) + \frac{1}{6} F[t_n, Y(t_n; r)]$$

$$\bar{Y}(t_{n+1}; r) = \bar{Y}(t_n; r) + \frac{1}{6} G[t_n, Y(t_n; r)]$$

The proposed Runge-Kutta approach (by MATLAB) is depicted in the Figure 1.

## 3. RESULT AND DISCUSSION

**Appllication in Radio Nuclides :** Let be considered a first-order ordinary differential equation

$$y'(w) = -x.y(w), y(w_0) = y_{0,} w \in I = [w_0, a]$$

Where x stands for the decay constant, $y_{0,}$ is the quantity of radionuclides in the mixture at the beginning of the operation, and y(w) is the quantity of radionuclides in each radioactive material. Given that nuclear disintegration is a stochastic process, the quantity of radionuclides can be unpredictable. Assuming that the starting value $y_{0,}$ in this scenario is unclear. However there are some circumstances in which it may not be known exactly how many radionuclides are in the radioactive material under investigation. . In this case, the starting value$y_0$ is regarded as an intuitionistic fuzzy number with a triangular form.

Let $x = 1$ , $I = [0, 1]$ and $y_0 = (5, 7, 9 ; 3, 7,11)$ . $(\alpha, \beta)$ – cut of $y(w_0) = y_0$ is given by:

$y(w_{0,r}) = y_0(r) = \{[y_\alpha, \overline{y_\alpha}], [y_\beta, \overline{y_\beta}]\}$ , $r \in [0, 1]$ and $0 \leq r = \alpha + \beta \leq 1$.

That means $y(w_{0,r}) = y_0(r) = \{[5 + 2\alpha, 9 - 2\alpha], [3 + 4\beta, 11 - 4\beta]\}$

, $r \in [0, 1]$ and $0 \leq r = \alpha + \beta \leq 1$.

In this problem Three methods can be used to approximate solutions for both membership and non-membership functions: the Runge-Kutta technique, the modified Euler method, and the Euler method.

### Case 1: (1, 2) Differentiability

Using equation (1.1) and the idea of (1, 2)-Differentiability, the following are the membership function's precise solutions:

$$y_\alpha(w) = (5 + 2\alpha)e^{-w}; \overline{y_\alpha} = (9 - 2\alpha$$

and the following provides the precise non-membership function solutions:

$$y_b(w) = (3 + 4b)e^{-w}; \overline{y_B(w)} = (11 - 4B)e^{-w}$$

**Table 1** displays the total error between the approximate and exact solutions for the membership function at various r-levels.

**Table. 1. Absolute Error for Membership function**

| r | Error by Euler Method | Error by Modified Euler Method | Error by Runge-Kutta Method |
|---|---|---|---|
| 0.0 | 0.40901574477 | 0.0016303928 | 8.3048E-06 |
| 0.22 | 0.30955259778 | 0.0013343142 | 6.6049E-06 |
| 0.42 | 0.2094894484 | 0.0010382356 | 5.0042E-06 |
| 0.61 | 0.2063814014 | 0.0009461612 | 4.6046E-06 |
| 0.81 | 0.2065814015 | 0.00026161 | 4.6045E-06 |
| 1.0 | 0.2062814014 | 0.0001261612 | 4.6045E-06 |

The total difference between the exact and approximate solutions for the non-membership function at different r-levels is shown in Table 2.

| Table. 2.Absolute Error for Non - Membership function | | | |
|---|---|---|---|
| **r** | **Error by Euler Method** | **Error by Modified Euler Method** | **Error by Runge-Kutta Method** |
| 0.0 | 0.2643814016 | 0.30275884 | 0.309390333 |
| 0.22 | 0.2642814015 | 0.242807073 | 0.247332268 |
| 0.42 | 0.2644814015 | 0.181855305 | 0.185374201 |
| 0.61 | 0.2641814014 | 0.122903536 | 0.123316134 |
| 0.81 | 0.2641814014 | 0.062951768 | 0.061358067 |
| 1.0 | 0.2642814014 | 0.003261612 | 4.6634E-06 |

## Case 2: (2, 1) Differencing

Equation (1.1) can be solved precisely for the membership function by using the concept of (2, 1)-Differentiability. The solutions are as follows:

$$y_{\alpha}(w) = (5 + 2\alpha)e^{-w}; \overline{y_{\alpha}} = (9 - 2\alpha$$

and the exact solutions of non-membership function are given by:

$$y_{b}(w) = (3 + 4b)e^{-w}; \overline{y_{B}(w)} = (11 - 4B)e^{-w}$$

The total difference at different r-levels between the exact and approximate membership function solutions is shown in Table 3.

| Table 3. Absolute Error for Membership function | | | |
|---|---|---|---|
| **r** | **Error by Euler Method** | **Error by Modified Euler Method** | **Error by Runge-Kutta Method** |
| 0.0 | 0.2162814016 | 0.1953379421 | 0.1531895167 |
| 0.22 | 0.2162814021 | 0.1923903536 | 0.1241916134 |
| 0.42 | 0.2162814015 | 0.0993427653 | 0.0931937101 |
| 0.61 | 0.2162814014 | 0.0963951768 | 0.06473958067 |
| 0.81 | 0.2162814015 | 0.0933475884 | 0.10333979033 |
| 1.0 | 0.2162814014 | 0.0903261612 | 4.3636E-06 |

Table 4 displays the overall error of the approximate and exact solutions for the non-membership function at various r-levels.

| Table 4. Absolute Error for Non-Membership function | | | |
|---|---|---|---|
| **r** | **Error by Euler Method** | **Error by Modified Euler Method** | **Error by Runge-Kutta Method** |
| 0.0 | 0.1993314947 | 0.034607855 | 1.6175E-05 |
| 0.22 | 0.1793051957 | 0.024886284 | 1.134E-05 |
| 0.42 | 0.1593788968 | 0.024164713 | 1.100E-05 |
| 0.61 | 0.1393525979 | 0.014443142 | 6.269E-06 |
| 0.81 | 0.1263814014 | 0.004261612 | 4.266E-06 |
| 1.0 | 0.1263814014 | 0.004261612 | 4.266E-06 |

## Case 3: (1, 1) Differencing

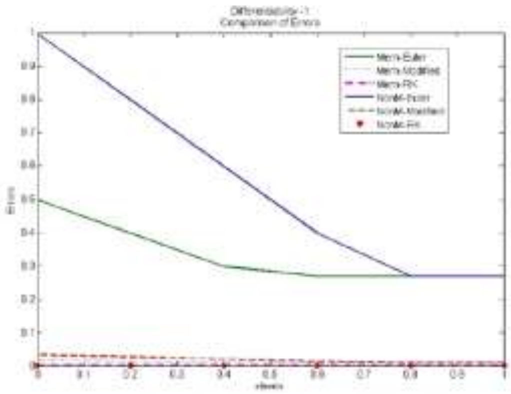
Equation (1.1) can be solved precisely for the membership function using the concept of (1, 1)-Differentiability using the following formula:

$$y_{\alpha}(w) = (5 + 2\alpha)e^{-w}; \overline{y_{\alpha}} = (9 - 2\alpha)$$

and the specific responses for the non-membership function are as follows:

$$y_b(w) = (3 + 4b)e^{-w}; \overline{y_B(w)} = (11 - 4B)e^{-w}$$

The errors made by the numerical methods described in this article are contrasted in the following graphic. Data was imported into MATLAB (Version R2021) to construct the figure.



**Case 4: (2, 2) Differentiability**

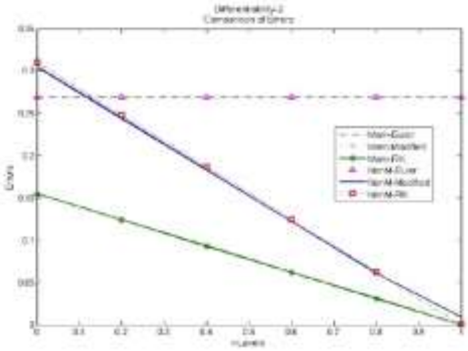
The exact answers of the membership function are given by:

$$y_\alpha(w) = (5 + 2\alpha)e^{-w}; \overline{y_\alpha} = (9 - 2\alpha)$$

and the precise responses for the non-membership function are as follows:

$$y_b(w) = (3 + 4b)e^{-w}; \overline{y_B(w)} = (11 - 4B)e^{-w}$$

An example of comparing the errors between the accurate and approximate solutions for the membership and non-membership functions may be seen in the figure below.



It is evident that the lengths of the supports of the equation (4.1) solutions under (1,1)-Differentiability, (1,2)-Differentiability, and (2,1)-Differentiability will all increase as the independent variable "w" rises. This indicates that as time passes, the system's radioactivity increases and its.

There could even be a negative radionuclide population. Nonetheless, it is commonly recognized that a material's radioactivity never increases above zero and always decreases with time. Thus, for problems of this kind, (2, 2)-Differentiability makes sense. The numerical solutions to equation (1.1) obtained by the Runge-Kutta method are substantially superior to those obtained by the other two methods in each of the four cases. However, by utilizing a smaller minimum step size, the inaccuracy could be reduced.

**Application for COVID-19**

Figures 2.3 and 2.4 show that the number of affected individuals was at least 200,000, reaching a high on Day 20. After that, the curvature started to progressively flatten. The government's protective measures, which mandate that everyone exercise social distancing and isolation at home, are most likely to blame for this. An increase in the number of impacted individuals who recovered from COVID-19 is seen in Figures 2.5 and 2.6. It is estimated that 400000 people are still alive. This could be as a result of the fact that more infected individuals are treated in quarantine centers with isolation and other therapies.

Figures 2.7 and 2.8 display the graph of all three SIR model classes, with starting parameters of 1.63 10-7 and 0.125, respectively, indicating that the immunization has not yet been administered in this simulation. The basic reproduction number (R_0) for the simulation model seen in Figures 2.7 and 2.8 is provided below. Consequently, we can state that the output computation fits the Euler's technique with SIR model flawlessly.

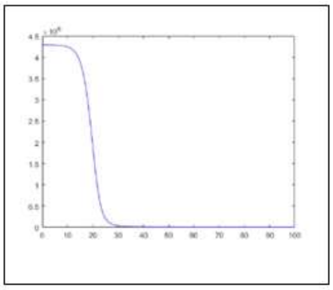


Figure 1

Simulation of Susceptible (S) in Euler's calculation of population

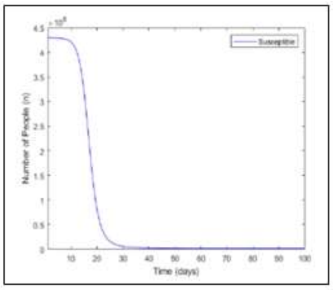



Figure 2.

Simulation of Susceptible (S) in SIR calculation of population

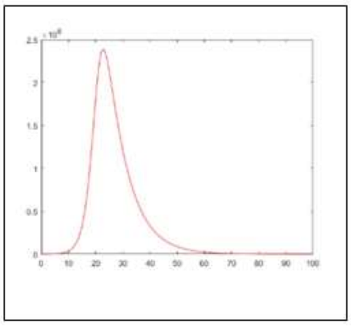


Figure 3

Simulation of Infected (I) in Euler's calculation of population

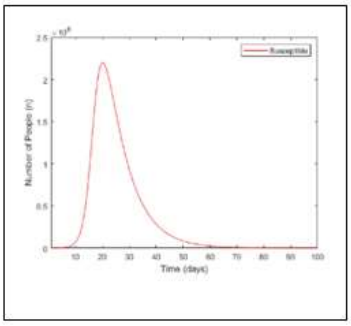



Figure 4

Simulation of Infected (I) in SIR calculation of population

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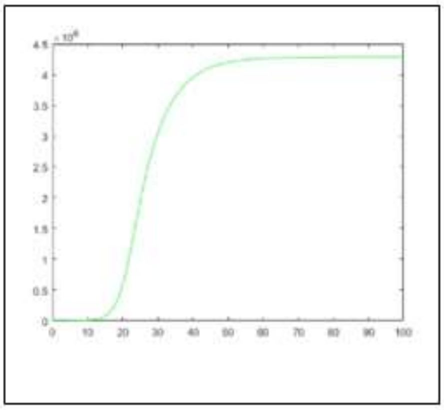

Figure 5

Simulation of Recovered (I) in Euler's calculation of population

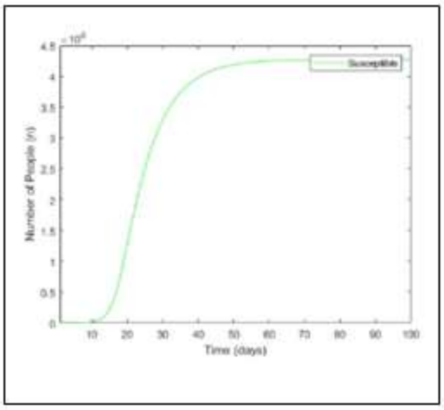


Figure 6

Simulation of Recovered (I) in SIR calculation of population

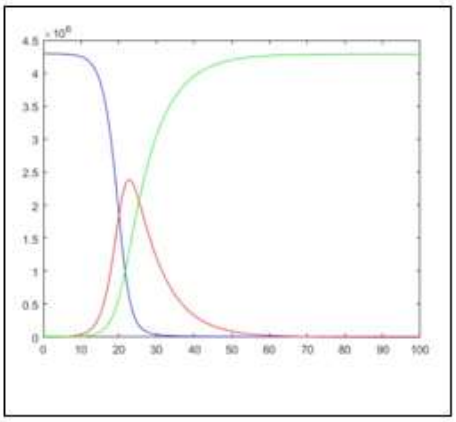

Figure 7

Simulation of the Susceptible (S), Infected (I), Recovered (R) in Euler's

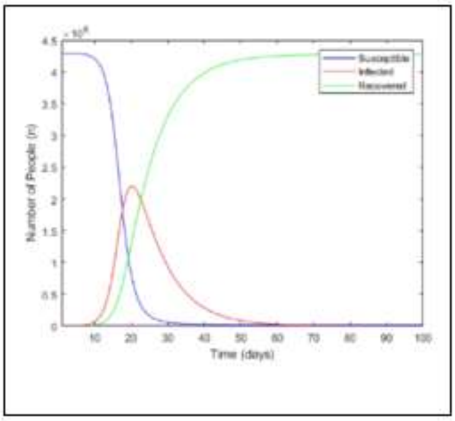


Figure 8

Simulation of the Susceptible (S), Infected (I), Recovered (R) in SIR

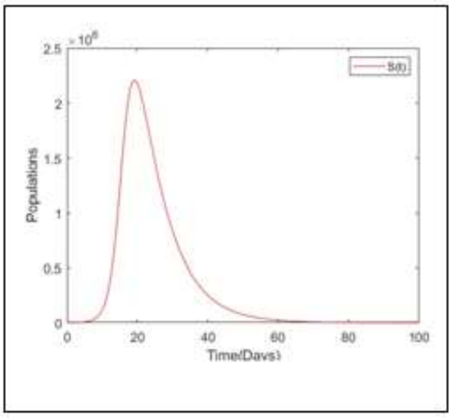


Figure 9

Simulation of Recovered (I) in Runge Kutta fourth order calculation of population

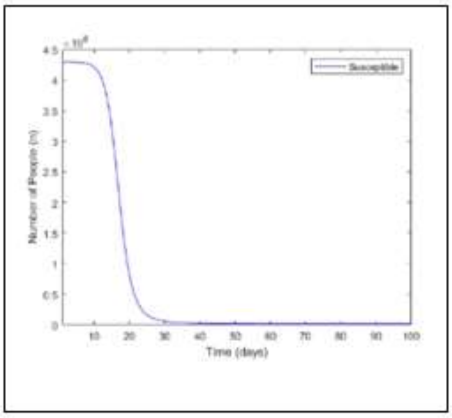


Figure 10

Simulation of Recovered (I) in Runge Kutta fourth order calculation of population

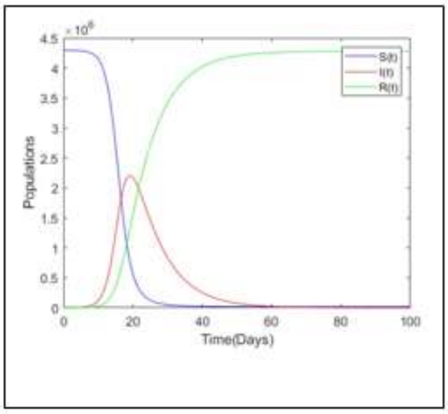


Figure 11

Simulation of the Susceptible (S), Infected (I), Recovered (R) in Runge

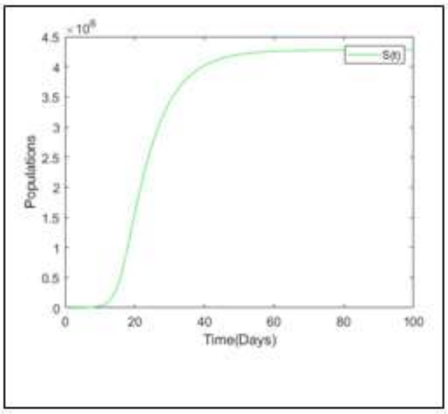


Figure 12

Simulation of Recovered (I) in Runge Kutta fourth order calculation of population

## 4. CONCLUSION

Through this article we apply the sol of Runge-Kutta method of order (2 , 3 , 4 , 5 and 6) utilizing a modified 2-step Simpson technique to numerical method of FDEs. We have ranking of the best to least (Rung Kutta of order six , Rung Kutta of order five, Rung Kutta of order four, Rung Kutta of order three, Rung Kutta of order two and a modified of 2-step Simpson) respectively. The researcher attempted to apply some of the problems in physics and medicine that the Runge-Kutta mothed was used to solve.

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