Bilangan : Jurnal Ilmiah Matematika, Kebumian dan Angkasa
Vol. 2 No. 4 Agustus 2024
e-ISSN :3032-7113; p-ISSN :3032-6389, Hal 262-274
DOI : https://doi.org/10.62383/bilangan.v2i4.203
Available online at: https://journal.arimsi.or.id/index.php/Bilangan

Enhancement of the Supplementary Method for Solving Problems in Partial Linear Programming

Dina Saad Faraj*
Department of Automation and Artificial Intelligence Engineering, College of Information Engineering, Al-Nahrain University, Jadriya, Baghdad, Iraq

*Corresponding Author: dina.saad@nahrainuniv.edu.iq**

Abstract . *The presented paper introduces an enhanced approach to find the available solution(s) for fractional linear programming problems (FLPP). While the common approach relies on considering the elements at the numerator and denominator to construct tables D_1 and D_2, followed by finding theirs and the Ze values, the new approach could be used to minimize the tables and decrease the calculation times and complicity.*

Keyword: *Supplementary Method, Partial LPP. Fractional Linear Programming Problems FLPP*

1. INTRODUCTION

The restrictions are then linear if the objection function is two (LF) ratio, and the variables are positive, so this matter is called (FLPP). The author presented a system under the name (FLPP) in his article. The method of approximating the objection function is one method to figure out the solution to the problems of (FP) in operations analysis. The solving method, the step method and the approximation of the objection function, however, initially depend on the objection function as the primary solution and then, in the math model, compensate for this solution.

Fractional Programming Problem can be applied using the following model:

$$Max\ Z = \frac{\sum_{i=1}^{n} C_i x_i +\ \alpha}{\sum_{i=1}^{n} D_i x_i +\ \beta}$$

According to restrictions:

$$Ax_i \ \leq\ b \qquad (1.1)$$

$$x_i\ \geq 0, \qquad i = 1,2, \dots, n$$

Where: x_i: are the targeted Linear variables in addition to the math module bounds.

C_i and D_i : are the processes of variables at the linear numerator and denominator targets, respectively.

$\alpha\ and\ \beta$: Are absolute constants for the linear numerator and denominator, respectively.

A : The matrix coefficients for variables beyond the process boundaries.

B: The fixed number (right side) of the phase boundaries.

Received: Juni 22, 2024; Revised: Juli 21, 2024; Accepted: Agustus 23, 2024; Published Agustus 25, 2024

The fractional linear programming concerns can be solved using the complementary process. It is performed by coding via joining the numerator linear variables.

On the opposite signal, while setting the denominator linear variables with code D_2 on the opposite signal, it is added to the simplified method table on the line ($M + 1$) and it is added to the simplified method table on the line ($M + 2$) when the logical equation boundaries for m are from the numbers, when the target line is from the numbers.

$$\boldsymbol{Ze = Z_2 D_1 - Z_1 D_2}$$

Hence:

Z_1: After being paid, the value of the numerator equation results in the values x.

Z_2: After being paid, the value of the denominator equation results in the values x.

If the optimal solution is obtained, the solving procedures are continued.

Utilization of the complementary method for FLPP to design the algorithm solution:

$\boldsymbol{Step}$ $(\mathbf{1})$ The Target Linear Process (*Ze*), which can be calculated using the following equation, are used to form the first line of the solution table (the first table in the simplified method):

$$\boldsymbol{Ze = Z_2 D_1 - Z_1 D_2}$$

Hence

Z_1 and Z_2: are the linear values of the nominator and the denominator equation after reimbursement and compensation of the corresponding variable x, respectively.

$\boldsymbol{Step}$ $(\mathbf{2})$ For neglected variables, the lines (2 – to – m) represent the number of bounds included in the mathematical module. (1.1) ($S_1, S_2, \dots . S_m$).

$\boldsymbol{Step}$ $(\mathbf{3})$ Line $1 + m$ is the nominator target Linear processes values multiplied by negative signal, which forms the value D_1.

$\boldsymbol{Step}$ $(\mathbf{4})$ Line $2 + m$ is the value of the denominator aim linear processes multiplied by the negative signal, representing the D_2 value.

$\boldsymbol{Step}$ $(\mathbf{5})$ Later, by a simpler approach, the problem is solved before the optimum solution is found and the value is removed. $x_1, x_2, \dots . x_n$.

$\boldsymbol{Step}$ $(\mathbf{6})$ Calculating the z value dependent on the equation below:

$$\boldsymbol{Z = \frac{Z_1}{Z_2}}, \quad \text{for the paper ([7]).}$$

See the following example for a further explanation:

Example

$Max\ Z = \frac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1}$

$S.T. \quad 5x_1 + 3x_2 \leq 15$

$5x_1 + 3x_2 \leq 10$

$x_1, x_2, x_3 \geq 0.$

Solution:

Setting the table of the first simplified method based on the algorithm of the complementary method, prior to determining the standard mathematical module formula for the problem, which results in:

$5x_1 + 3x_2 + S_1 = 15$

$5x_1 + 3x_2 + S_2 = 10$

$x_1, x_2, x_3 \geq 0.$

After Ze value is reached based on below equation:

$$\boldsymbol{Ze = Z_2 D_1 - Z_1 D_2}$$

The table of the first simplified method is filled as shown in below:

Table 1. Details for the solution

$\boldsymbol{Basic}$	$\boldsymbol{V}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$
$\boldsymbol{Ze}$	0	−5	−3	0	0
$\boldsymbol{S_1}$	15	3	5	1	0
$\boldsymbol{S_2}$	10	5	2	0	1
$\boldsymbol{D_1}$	0	−5	−3	0	0
$\boldsymbol{D_2}$	0	−5	−2	0	0

Then: $\boldsymbol{Z_1 = 0}$, $\boldsymbol{Z_2 = 1}$ $\boldsymbol{thus}$ $\boldsymbol{Z = 0}$, $C = 1$, $R = 2$.

Now, we can see from the figures (1 and 2) below all the condition variables in the table 1:

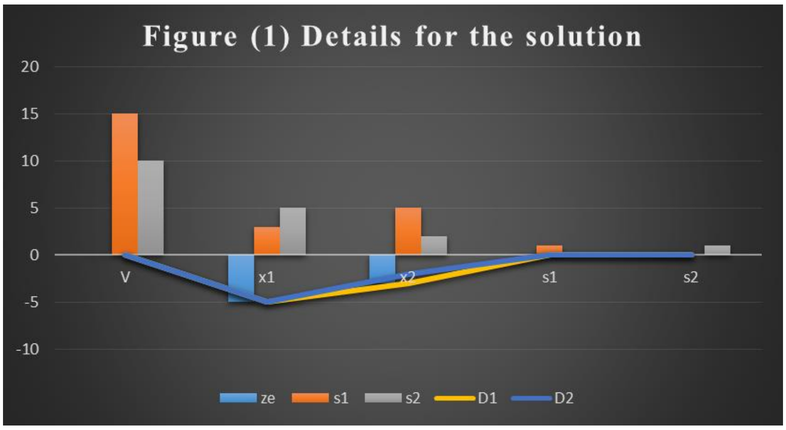

Figure (1) Details for the solution

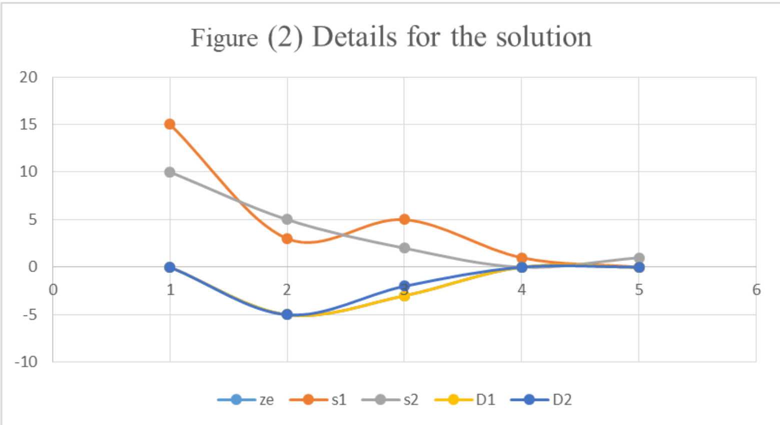

Figure (2) Details for the solution

And the figures above appears the control of the objective function in the solution (i.e. enhancement method).

Table 2. Details for the results

Basic	***V***	x_1	x_2	S_1	S_2
Ze	10	0	-11	0	1
S_1	9	0	3.80	1	-0.60
S_2	2	1	0.40	0	0.20
D_1	10	0	-1	0	1
D_2	10	0	0	0	1

Thus:

$$Z_1 = 10 \ , Z_2 = 11 \quad then \quad Z = 0.9090909, \quad C = 2, R = 1$$

Then, we can show from the figures (3 and 4) below all the condition variables in the table 2 are satisfied directly:

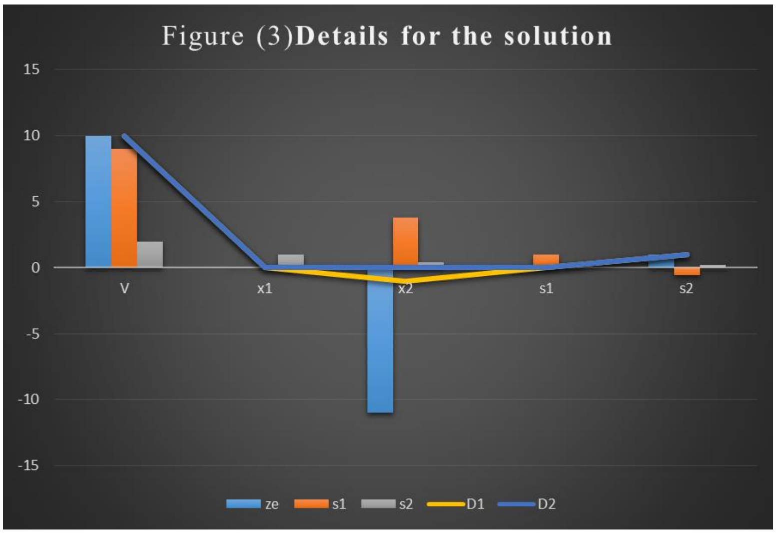

Figure (3)Details for the solution

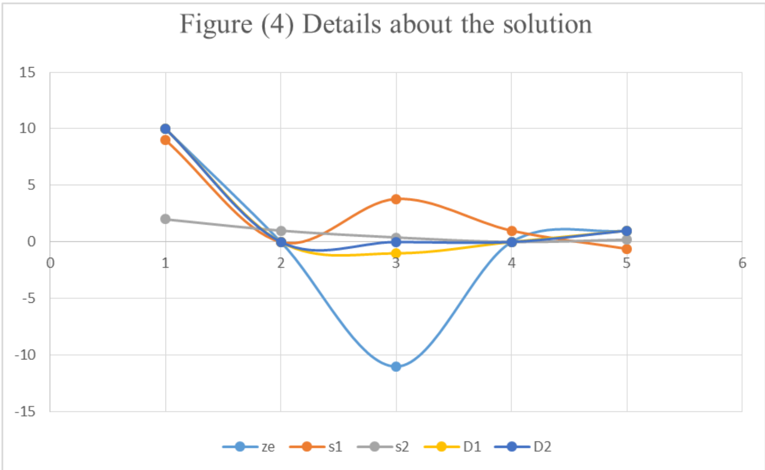

Figure (4) Details about the solution

We can see form figures (3 and 4) above all the variables comparative for the enhancement method are the best when applied it in the examples.

Table 3. Details for the present solution

Basic	***V***	x_1	x_2	S_1	S_2
Ze	12.37	0	0	2.89	-3.11
S_1	2.37	0	1	0.26	-0.16
S_2	1.05	1	0	-0.11	0.26
D_1	12.37	0	0	0.26	0.84
D_2	10	0	0	0	1

Then: $Z_1 = 12.36842$, $Z_2 = 11$ *thus*

$Z = 1.124402$

$C = 4$, $R = 2$.

Now from the figures (5 and 6) below we can see the following:

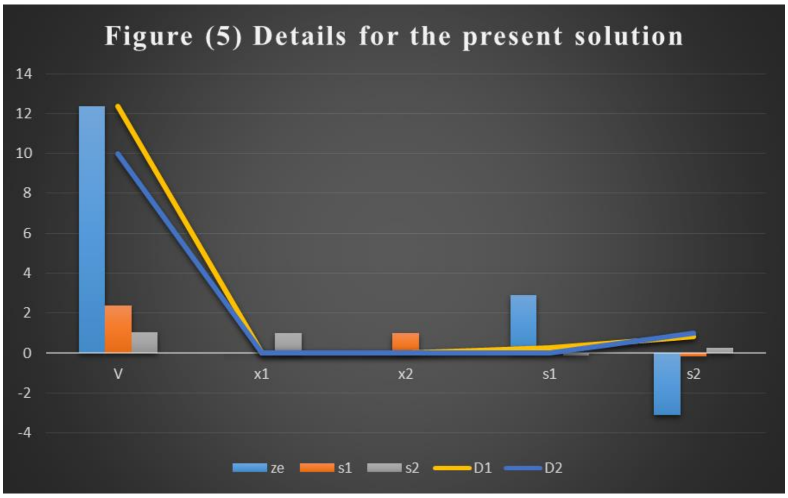

Figure (5) Details for the present solution

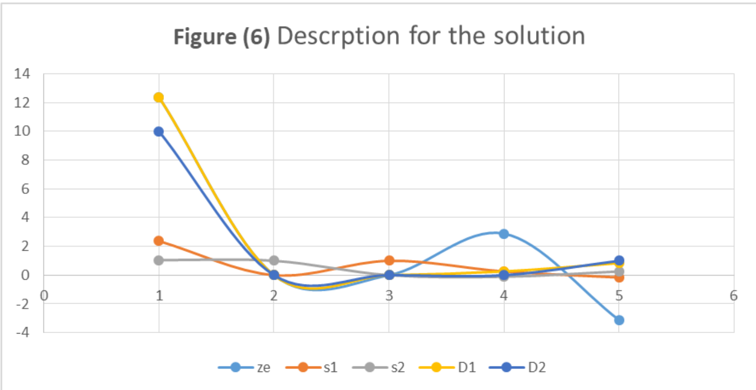

Figure (6) Descrption for the solution

From the figures (5 and 6) above we can see the objective function is the best and all variables comparative for the enhancement method are stable.

Table 4. Specifics for the solution

Basic	***V***	x_1	x_2	S_1	S_2
Ze	9	11.80	0	0.60	0
S_1	3	0.60	1	0.20	0
S_2	4	3.80	0	-0.40	1
D_1	9	-3.20	0	0.60	0
D_2	6	-3.80	0	0.40	0

The best-fit solution is: $\boldsymbol{x_1 = 0}$, $\boldsymbol{x_2 = 3}$, $\boldsymbol{Z_1 = 9}$, $\boldsymbol{Z_2 = 7}$

Then $\boldsymbol{Max\ Z = 1.285714.}$

Now from the figures (7 and 8) below we can see the following:

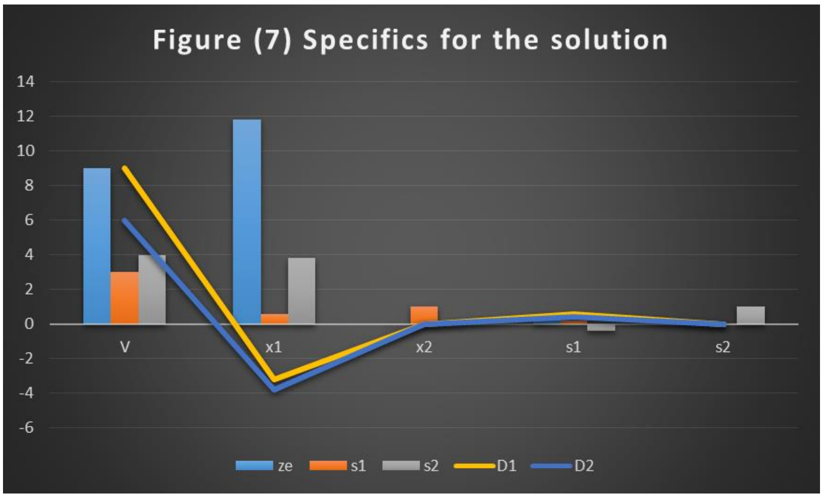

Figure (7) Specifics for the solution

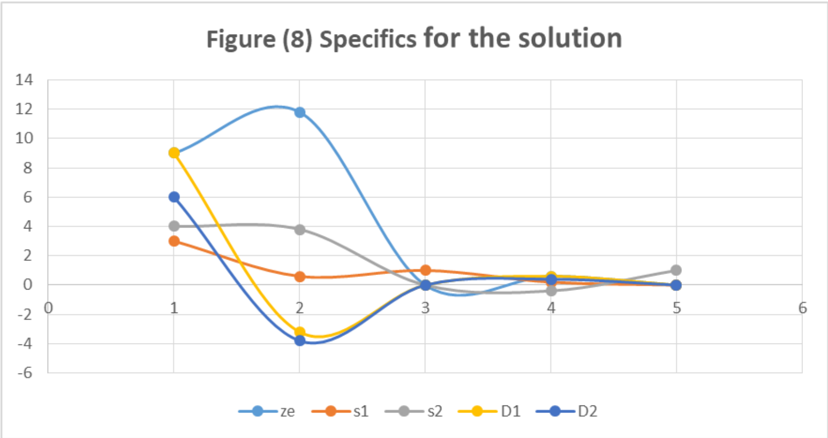

Figure (8) Specifics for the solution

The figures (7 and 8) above introduced all details about the enhancement method in the field of fractional problem.

Production Algorithm of the Supplementary Process to Solve (FLPP)

Step (1): Let's take the equation (1.1)

Step (2): To make the linear fractional (1.1) to its maximum limit, the linear nomirator to denominator ratio should be maximized by maximizing the linear numirator and minimizing the denominator. The aforementioned two values are coded to be Max Z1 and Min Z2 for the linear numirator and denominator, respectively.

Step (3): The linear denominator is converted from $Min\ Z_1\ to\ Max\ Z_2$ by negative signal multiplying the sides of the equations.

Step (4): The latest linear goal is as follows:

$$Max\ ZZ = Max\ Z_1 + Max\ Z_2$$

Step (5): Matching the bounds to the mathematical module.

Step (6): Arrange the main simpler table of approaches and limit: the pivot axis.

Step (7): At each table (key-solving variables), the corresponding x of variable values is taken and its value is compensated in the equation (1.1) and then derived by the Z value in the current row, followed by comparing Z to the previous table. The comparison returns two possible values greater or smaller than the previous solution. The greater value of current Z means that the solution can continue to be solved by canceling the following table before the required solution is found. When the current Z is less than the previous value, however, means that the x value in the previous table is the optimum solution, and the Z value is the best fit even if the optimal solution value is not shown in the solution table. The following example can be referenced to justify the method.

Example

As in example (1.2.1)

$Max\ Z = \frac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1}$

$$S.T. \quad 5x_1 + 3x_2 \leq 15$$

$$5x_1 + 3x_2 \leq 10$$

$$x_1, x_2 \geq 0.$$

Solution:

The nominator must be as high as possible and the denominator as low as possible in order to get the highest value:

$Max\ Z_1 = 5x_1 + 3x_2$

$S.T. \quad 3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$

$Min\ Z_2 = 5x_1 + 2x_2 + 1$

$S.T. \quad 3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$

Then:

$Max\ ZZ = Z_1 + Z_2$

$S.T. \quad 3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$

Thus we can get:

$Max\ ZZ = 5x_1 + 3x_2 - 5x_1 - 2x_2 - 1$

$S.T. \quad 3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$

The new mathematical module becomes:

$Max\ ZZ = x_2 - 1$

$S.T. \quad 3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$

We're translating the math module above to the regular formula:

$ZZ - x_2 = -1$

$S.T. \quad 3x_1 + 5x_2 + S_1 \leq 15$

$5x_1 + 2x_2 + S_2 \leq 10$

$x_1, x_2 \geq 0.$

The first table in the normal simplified method can now be arranged as in following:

Table 5. Details for the solution

$\boldsymbol{Basic}$	$\boldsymbol{V}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$
ZZ	-1	0	-1	0	0
S_1	15	3	5	1	0
S_2	10	5	2	0	1

$\boldsymbol{C = 2, \ R = 1}$ **and** $\boldsymbol{Max\ Z = -1.}$

Therefore, for the figures (9 and 10) below show that the dominance for the presented of enhancement method.

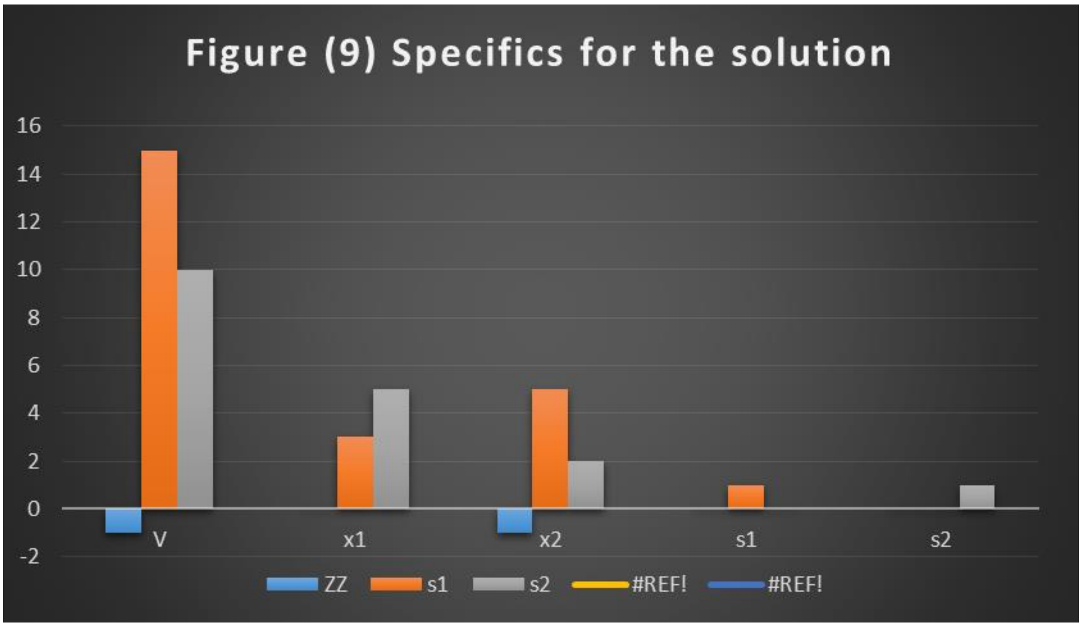

Figure (9) Specifics for the solution

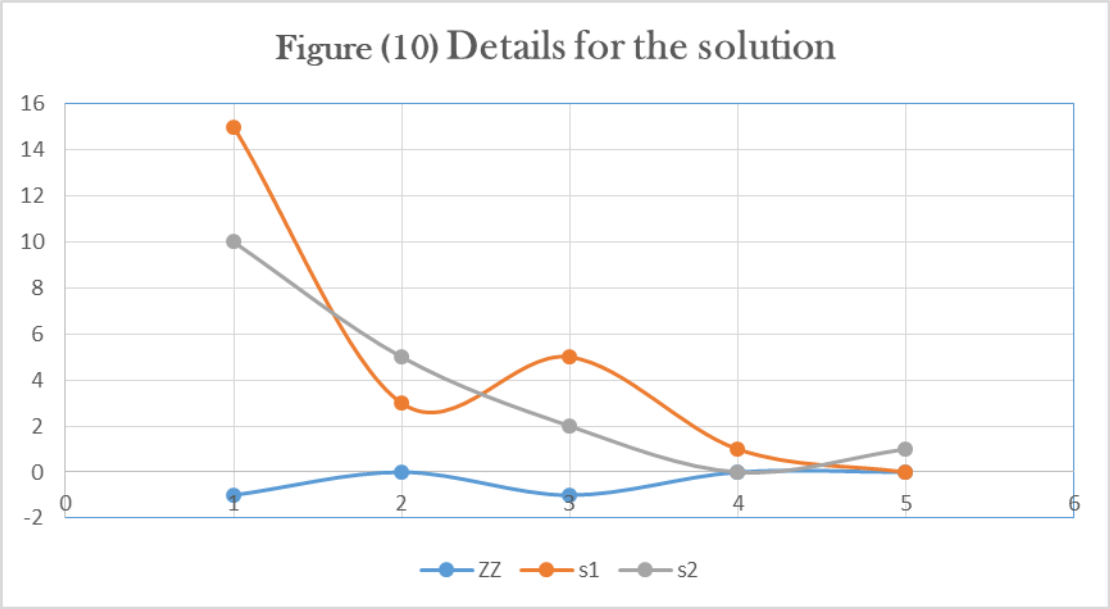

Figure (10) Details for the solution

Also, the figures (9 and 10) above showed that all details about the new method that is the best.

Table 6. Details for the solution

$Basic$	V	x_1	x_2	S_1	S_2
ZZ	2	0.60	0	0.20	0
x_2	3	0.60	1	0.20	0
S_2	4	3.80	0	-0.40	1

The optimal solution is: $x_1 = 0$, $x_2 = 3$. When compensating the values xi, we have: $Max\ Z = 1.285714$

Therefore, as long as the resulting Z value in the second table (2.2.2) is greater than the resulting Z value in the first table (2.2.1) and the solution reaches the optimal statue, then the perfect solution to this problem is: $x_1 = 0,\ x_2 = 3$

Therefore: $Max\ Z = 1.285714$

Therefore, from the figures (11 and 12) above we can see the all details about the enhancement method that it is performance when applied it in the field of fractional linear programming.

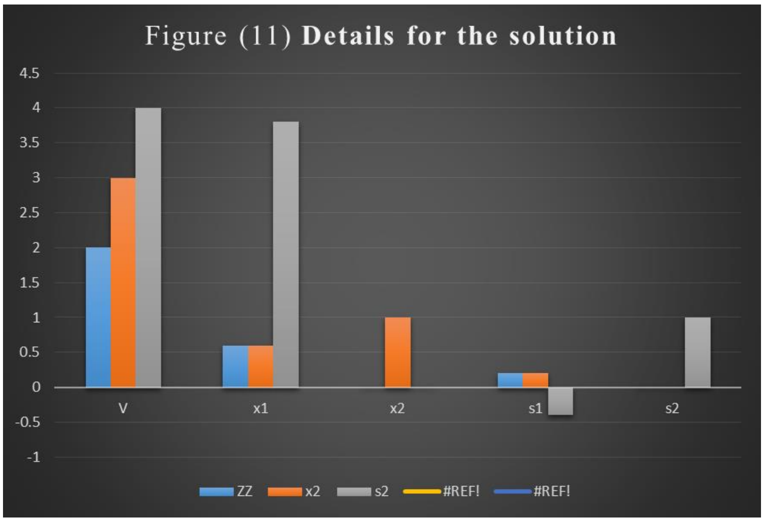

Figure (11) Details for the solution

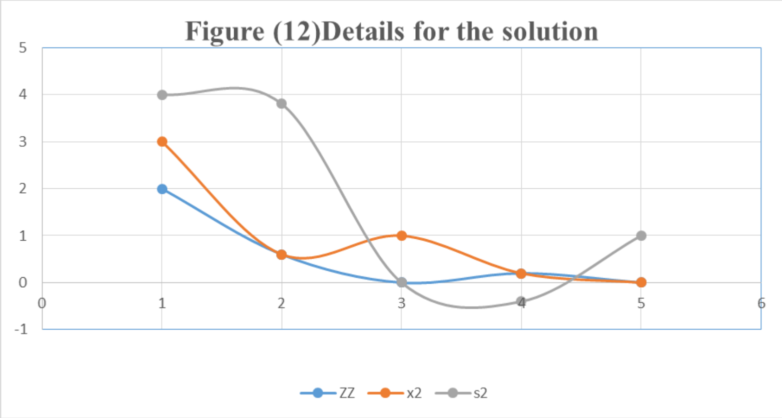

Figure (12)Details for the solution

2. CONCOLUTION

It was found that after comparing the two methods, the proven technique decreased the tables and solution steps, saving time, commitment and speeding up the solution. The paper proposes a modern improvement focused on Partial Linear Programming to overcome FFPs. In addition, from the numerical examples presented, we can prove that the new approach used in the solution was the best and showed its efficacy as well. The investigator should extend the fuzzy method to this scheme in order to present future research work. Also, in the applications, the best results were achieved using the enhancement method with an advantage in the computational time to the PLP approaches optimization results and an advantage to the current method.

REFERENCES

Bitran, G. R., & Novaes, A. G. (1973). Linear programming with a fractional objective function. University of Sao Paulo, Brazil, 21, 22–29.

Hassan, I. H., Mitilf, R. J., & Jaber, W. K. (2019). Development cutting plane method to find optimal integer solution for fuzzy linear programming problems. New Trends in Mathematical Sciences, 7(3), 350–355.

Jaber, W. K. (2017). Total penalty cost in the fuzzy schedule on parallel machines. International Journal of Mathematical Sciences and Engineering Applications, 11(2), 75–84.

Jaber, W. K. (2020). Solve bounded fractional linear programming by dynamic programming. Test Engineering and Management Journal, 83, 13158–13164.

Jaber, W. K. (2020). Solve fractional linear programming by using branch and bound method. Test Engineering and Management Journal, 83, 13165–13171.

Jaber, W. K., & Hassan, A. A. (2009). The development method of approximation the objective function to solve fractional L.P. Journal of Al-Anbar University for Pure Science, 3(3).

Lenstra, K. (2001). Fractional linear programming, linear time dynamic-programming, algorithms for new classes of restricted TSPS: A computational study. Journal on Computing, 13(1).

Swarup, K. (1962). Linear fractional programming. Operations Research, 10, 380–387.