Bilangan : Jurnal Ilmiah Matematika, Kebumian dan Angkasa
Vol. 2 No. 4 Agustus 2024
e-ISSN :3032-7113; p-ISSN :3032-6389, Hal 11-23
DOI: https://doi.org/10.62383/bilangan.v2i4.136

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An Investigation Of A New Hybrid Conjugates Gradient Approach For Unconstrained Optimization Problems

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Abstract. *This work introduces a novel hybrid conjugate gradient (CG) technique for tackling unconstrained optimisation problems with improved efficiency and effectiveness. The parameter θ_k is computed as a convex combination of the standard conjugate gradient techniques using β_k^{LS} and β_k^{Ha}. Our proposed method has shown that when using the strong Wolfe-line-search(SWC) under specific conditions, it achieves global theoretical convergence. In addition, the new hybrid CG approach has the ability to generate a search direction that moves downward with each iteration. The quantitative findings obtained by applying the recommended technique about 30 functions with varying dimensions clearly illustrate its effectiveness and potential.*

Keywords: *Numerical optimization, Unconstrained objective function, Hybrid gradient methods, Global convergence, Numerical experiment*

1- INTRODUCTION

Assuming we have a general function $f: R^n \to R$ that is continuously differentiable. Also, examine the subsequent un-constrained optimisation issue

$$\min\,(or\ max)\,\{f(x)\} \in\ R^n \tag{1}$$

The space R^n represent an "n-dimensional Euclidean space".

To solve Equation (1), we begin by selecting an initial guess $x_0 \in\ R^n$. We then employ a non-linear conjugate gradient approach to produce a series $\{x_k\}$

$$x_{k+1} = x_k + \alpha_k\, d_k \qquad ,k = 0,\ \ 1,\ 2, \ldots \tag{2}$$

The value of $\alpha_k > 0$ is determined by a process called "line search". The direction denoted d_k are formed using a specific design as

$$d(x) = \begin{cases} -g_k, & k = 0 \\ -g_k + \beta_k d_k, & k > 0 \end{cases} \tag{3}$$

Let g_k be the gradient of $f(x_k)$, and β_k be a scalar parameter that defines the properties of conjugate gradient techniques.

In the field of computing, the step-size α_k is considered to meet any of the line search conditions during the procedure. This work focuses on the robust Wolfe line search Equation (4) [1].

Received: Mei 31, 2024; Accepted: Juni 24, 2024; Published: Agustus 31, 2024
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$$f(x_k + \alpha_k d_k) \leq f(x) + \delta \alpha_k g_k^T d_k \quad , \qquad 0 \leq \delta \leq \frac{1}{2} \qquad (4)$$

$$|d_k^T g(x_k + \alpha_k d_k)| \leq -\sigma g_k^T d_k, \qquad 0 \leq \delta \leq 1$$

The search direction, denoted as d_k is explicitly defined in Equation (3).

Researchers have devoted significant attention to CG approaches for a long time. The result of such investigations is the development of numerous formulas with variations in the CG coefficient (β_k) for addressing unconstrained optimisation issues[2].

Here are several typical formulas for β_k:

$$\beta_k^{ER} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}, \qquad \text{FR(Fletcher – Reeves) [3]}$$

$$\beta_k^{PR} = \frac{g_{k+1}^T y_k}{g_k^T g_k}, \qquad \text{PR (Polak – Ribiere) [4]}$$

$$\beta_k^{DY} = \frac{g_k^T g_k}{d_{k-1}^T y_{k-1}}, \qquad \text{DY (Dai – Yuan) [5]}$$

$$\beta_k^{CD} = \frac{-g_k^T g_k}{d_{k-1}^T g_{k-1}}, \qquad \text{CD (conjugate descent) [6]}$$

$$\beta_k^{LS} = \frac{-g_k^T y_{k-1}}{d_{k-1}^T g_{k-1}} \qquad \text{LS (Liu – Storey) [7]}$$

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{d_{k-1}^T y_{k-1}} \qquad \text{HS(Hestenes- Stiefel) [8]}$$

Were

$$y_{k-1} = g_k - g_{k-1}, \qquad (5)$$

Despite their excellent global theroritic convergence, the computational performance of the CG approaches β_k^{LS} *and* β_k^{CD} is lower. however, superior computing performance is generally achieved by the $\beta_k^{PR}, \beta_k^{LS}$ and β_k^{HS}, despite the fact that they haven't demonstrated convergence[9]. "In most cases, hybrid conjugate gradient methods are more efficient than basic conjugate gradient methods. The hybrid conjugates gradient techniques discussed in this study are of particular importance. These algorithms are a mixture of a number of different conjugate gradient techniques."[10]

The main principle behind their strategy is to capitalize on projected outcomes. They are frequently promoted as a means to prevent congestion. We have presented a novel hybrid CG approach that relies on the Ha and LS methods. The method incorporates the parameters β_k^{Ha} and β_k^{LS}.

$$\beta_k^{LS} = \frac{g_{k+1}^T y_k}{-d_k^T g_k} \quad , \quad \beta_k^{Ha} = \frac{\|g_{k+1}\|^2}{(f_{k+1} - f_k/\alpha k - 3/2) d_k^T g_k} \qquad [11].$$

To solve the unconstrained optimization problems with β_k^{MR}

The parameter β_k^H in our proposed method is computed as a convex combination of β_k^{Ha} *and* β_k^{LS} such that

$$\beta_k^{MR} = (1 - \theta_k)\beta_k^{LS} + \theta_k \beta_k^{Ha} \qquad (6)$$

We have $0 \le \theta_k \le 1$

The rest of the document is structured in the following manner. In section 2, we outline our suggested approach for acquiring the parameter θ_k using several methodologies. We further analyse the adequate descent property of our approach under certain suitable conditions, and moreover establish the parameter constraint in the form of $0 \le \theta_k \le 1$. Section 3 encompasses several assumptions, whereas section 4 defines the global convergence of the proposed approach. Section 5 concludes by presenting the results of numerical experiments that were conducted.

2. GRADIENT METHOD CONJUGATES WITH THE NEW HYBRID

2.1 The New θ_k Parameter Derivation

The recurrence is utilized to determine the iterates $x_0, x_1, x_2, \dots\dots$ of our method (2.2). The strong Wolfe requirements (4) determine the step size $\alpha_k > 0$, whereas the rule generates the directions.

$$\begin{Bmatrix} d_0 = -g_0 \\ d_{k+1} = -g_{k+1} + \beta_k^H d_k \end{Bmatrix} \qquad (7)$$

and the parameter β_k^H in the form Eq(6), where $0 \le \theta_k \le 1$ and we derived hybrid parameter by, derivation of the new parameter

$$B_k^{H_1} = (1-\theta_k)B_k^{LS} + \theta_k B_k^{Ha}$$

$$B_k^{H_1} = (1-\theta_k)\frac{g_{k+1}^T y_k}{-d_k^T g_k} + \theta_k \frac{\|g_{k+1}\|^2}{(f_{k+1}-f_k)/\alpha_k - 3/2\, d_k^T g_k}$$

Derivation of the new hybrid parameter from the above equation

$$d_k = \{\begin{matrix} -g_k & if \quad k=0 \\ -g_k + B_k^{H_1} d_{k-1} & if \quad k>0 \end{matrix}$$

since $0 \le \theta_k \le 1$

$$d_{k+1} = -g_{k+1} + (1-\theta_k)\frac{g_{k+1}^T y_k}{-d_k^T g_k}\, d_k + \theta_k \frac{\|g_{k+1}\|^2}{(f_{k+1}-f_k)/\alpha_k + 3/2\, d_k^T g_k}\, d_k$$

Wolf's situation makes that imperative

$$f_{k+1} \le f_k + \delta\, \alpha_k\, g_k^T d_k \ , \qquad 0 \le \delta \le \frac{1}{2}$$

Multiply both sides of the equation y_k^T

$$y_k^T d_{k+1} = -y_k^T g_{k+1} + (1-\theta_k)\frac{g_{k+1}^T y_k}{-d_k^T g_k}\, y_k^T\, d_k + \theta_k \frac{\|g_{k+1}\|^2}{\frac{\delta\, \alpha_k\, g_k^T d_k}{\alpha_k} - \frac{3}{2} d_k^T g_k}\;\; y_k^T d_k$$

It's essential the $y_k^T d_{k+1} = 0$ by perry, then

$$0 = \; -y_k^T g_{k+1} + (1-\theta_k)\frac{g_{k+1}^T\, y_k}{-d_k^T g_k}\, y_k^T\, d_k + \theta_k \frac{\left\|g_{k+1}\right\|^2}{-\delta_1 g_k^T d_k}\;\; y_k^T d_k$$

$$0 = \; -y_k^T g_{k+1} - \frac{g_{k+1}^T\, y_k}{d_k^T g_k}\, y_k^T\, d_k + \theta_k\, \frac{g_{k+1}^T\, y_k}{d_k^T g_k}\, y_k^T\, d_k - \theta_k \frac{\left\|g_{k+1}\right\|^2}{\delta_1 g_k^T d_k}\;\; y_k^T d_k$$

$$y_k^T g_{k+1} + \frac{g_{k+1}^T\, y_k}{d_k^T g_k}\, y_k^T\, d_k = \theta_k [\frac{g_{k+1}^T\, y_k}{d_k^T g_k}\, y_k^T\, d_k - \frac{\|g_{k+1}\|^2}{\delta_1 g_k^T d_k}\;\; y_k^T d_k]$$

$$\therefore \theta_k = \frac{y_k^T g_{k+1} + \frac{g_{k+1}^T\, y_k}{d_k^T g_k}\, y_k^T\, d_k}{\frac{g_{k+1}^T\, y_k}{d_k^T g_k}\, y_k^T\, d_k - \frac{\|g_{k+1}\|^2}{\delta_1 g_k^T d_k}\;\; y_k^T d_k}$$

$$\theta_k = \frac{\frac{y_k\, g_{k+1}}{y_k^T\, d_k} + \frac{g_{k+1}^T\, y_k}{d_k^T\, g_k}}{[\frac{g_{k+1}^T\, y_k}{d_k^T g_k} + \frac{\left\|g_{k+1}\right\|^2}{(\delta_{\alpha k} g_k^T d_k/\alpha_k - 3/2\, d_k g_k^T)}]} \tag{8}$$

In order to Prove that $0 \leq \theta_k \leq 1$ we will use the following mathematical lemma.

Lemma: The hybridization parameter in the convex structure is limited to 0 and 1. And so is our hybridization scalar Eq(8).

Proof

1-The first possibility:- we take the case of a fraction that is less than zero. we get a contradiction because a positive value + a positive value is impossible for a negative result to appear.

$$\frac{Numerator + denominator}{Numerator + the\;\; vule}$$

$$\frac{\frac{y_k\, g_{k+1}}{y_k^T\, d_k} + \frac{g_{k+1}^T\, y_k}{d_k^T\, g_k}}{[\frac{g_{k+1}^T\, y_k}{d_k^T g_k} + \frac{\|g_{k+1}\|^2}{(\delta_{\alpha k} g_k^T d_k/\alpha_k - \; 3/2\, d_k g_k^T)}]} \qquad < \frac{0}{1}$$

The product of two sides multiplied by two means.

$$\frac{y_k\, g_{k+1}}{y_k^T\, d_k} + \frac{g_{k+1}^T\, y_k}{d_k^T\, g_k}$$

A contradiction because a positive value + a positive value

2-The second possibility: we take, the fraction greater than zero and obtain that the denominator is greater than zero

$$0 < \frac{\frac{y_k\, g_{k+1}}{y_k^T\, d_k} + \frac{g_{k+1}^T\, y_k}{d_k^T\, g_k}}{\left[\frac{g_{k+1}^T\, y_k}{d_k^T g_k} + \frac{\|g_{k+1}\|^2}{(\delta_{\alpha k} g_k^T d_k/\alpha_k - \; 3/2\, d_k g_k^T)}\right]}$$

3-The third possibility:- we take the fraction less than one and obtain that the denominator is the greater than the numerator

$$\frac{\frac{y_k\, g_{k+1}}{y_k^T\, d_k} + \frac{g_{k+1}^T\, y_k}{d_k^T\, g_k}}{\left[\frac{g_{k+1}^T\, y_k}{d_k^T g_k} + \frac{\|g_{k+1}\|^2}{(\delta_{\alpha k} g_k^T d_k/\alpha_k - \; 3/2\, d_k g_k^T)}\right]} < 1$$

$$\frac{the\ vlue + denominator}{Numerator + the\ vlue} < 1$$

$$the\ vlue + denominator\ < the\ vlue\ + Numerator$$

$$denominator\ < Numerator$$

$$\frac{y_k\, g_{k+1}}{y_k^T\, d_k} + \frac{g_{k+1}^T\, y_k}{d_k^T\, g_k} < \frac{g_{k+1}^T\, y_k}{d_k^T\, g_k} + \frac{\|g_{k+1}\|^2}{(\delta_{\alpha k} g_k^T d_k/\alpha_k - \; 3/2\, d_k g_k^T)}$$

$$\frac{y_k\, g_{k+1}}{y_k^T\, d_k} < \frac{\|g_{k+1}\|^2}{(\delta_{\alpha k} g_k^T d_k/\alpha_k - \; 3/2\, d_k g_k^T)}$$

4- The fourth possibility:- we take the fraction greater than one and obtain a contradiction due to the presence of the squared swelling. By simplifying , this result appears (a contradiction)

$$1 < \frac{\frac{y_k\, g_{k+1}}{y_k^T\, d_k} + \frac{g_{k+1}^T\, y_k}{d_k^T\, g_k}}{\left[\frac{g_{k+1}^T\, y_k}{d_k^T g_k} + \frac{\|g_{k+1}\|^2}{(\delta_{\alpha k} g_k^T d_k/\alpha_k - \; 3/2\, d_k g_k^T)}\right]}$$

(contradiction) Due to the presence of $||g_{k+1}||^2$ square swelling , by simplifying we will obtain a (contradiction)

3. The Descending Characterized

Assume that the function fulfills the hypotheses, H and when α_k it satisfies the strong wolf line, then B_k^H it satisfies the formula, then the following is holding.

$$g_{k+1}^T d_{k+1} < 0 , \quad \forall \ k$$

Proof: using the principle of mathematical induction

$$d_{k+1}^T g_{k+1} = -g_{k+1}^T g_{k+1} + (1-\theta_k)\frac{g_{k+1}^T y_k}{-d_k g_k} d_k^T g_{k+1} + \theta_k \frac{||g_k+1||^2}{(\delta_{\alpha k} g_k^T d_{k/\alpha k-3/2}) d_k^T g_k} d_k^T g_{k+1}$$

$$y_k = g_{k+1} - g_k$$

$$||g_{k+1}||^2 = g_{k+1}^T g_{k+1} \because$$

$$= ||g_k+1||^2 - \frac{g_{k+1}^T(g_{k+1}-g_k)}{d_k^T g_k} d_k^T g_{\,k+1} + \theta_k \frac{g_{k+1}^T(g_{k+1}-g_k)}{d_k^T g_k} d_k^T g_{k+1} + \theta_k \frac{||g_{k+1}||^2}{(\delta_{\alpha k}-3/2) g_k^T d_k} d_k^T g_{k+1}$$

$\because 0 \le \theta_k \le 1$

$$= ||g_k+1||^2 - \frac{(g_{k+1}^T g_{k+1}) - (g_{k+1}^T g_k)}{d_k^T g_k} d_k^T g_{k+1} + \theta_k \frac{(g_{k+1}^T g_{k+1}) - (g_{k+1}^T g_k)}{d_k^T g_k} d_k^T g_{k+1} + \theta_k \frac{||g_{k+1}||^2}{(\delta-3/2) g_k^T d_k} d_k^T g_{k+1}$$

$$= ||g_k+1||^2 - \frac{||g_k+1||^2 - \psi||g_k+1||^2}{d_k^T g_k} - \sigma\, d_k^T g_k + \theta_k \frac{||g_k+1||^2 - \psi||g_k+1||^2}{d_k^T g_k} - \sigma\, d_k^T g_k + \theta_k \frac{||g_k+1||^2}{(\delta-3/2) g_k^T d_k} - \sigma\, d_k^T g_k$$

Use the (SWC) search $\quad \sigma d_k^T g_k \le d_k^T g_{k+1} \le -\sigma d_k^T g_k$

$$g_{k+1}^T g_k \le -\psi ||g_{k+1}||^2 \qquad \text{(AL-Bayat i\& Jameel-2014)}$$

$$= ||g_k+1||^2 - ||g_k+1||^2 - \psi||g_k+1||^2 - \sigma + \theta_k||g_k+1||^2 - \psi||g_k+1||^2 - \sigma + \theta_k \frac{||g_k+1||^2}{(\delta-3/2)} - \sigma$$

$$d_{k+1} g_{k+1} \le -[1-1-\psi-\sigma\theta_k - \sigma + \frac{\theta_k}{(\delta-3/2)} - \sigma]\ ||g_k+1||^2$$

$$\because c_1 = -\left[1-1-\psi-\sigma\theta_k-\sigma+\frac{\theta_k}{(\delta-3/2)}-\sigma\right] \qquad 0 < c_1 < 1$$

$$d_{k+1} g_{k+1} \le -c_1\, g_{k+1} g_{k+1}$$

4. Global Convergence

f A uniformly convex function

$\lim_{k\to\infty} inf \|g_k\| = 0$

$0 \leq \theta \leq 1 \because$

it is obtained from the strong wolff line $\|\delta_k\|$ approaching zero, as there is an on-negative number $x_k > 0$ so that

$$\|g_k\|^2 \geq n_1\|\delta_k\|^2$$

$$\|g_{k+1}\|^2 \geq n_1\|\delta_k\|^2$$

Since $d_k$the convergence vector and obtain $\alpha k > 0$ is the step length, we through the strong wolff line if

$$\sum_{k\geq 1} \frac{1}{\|d_{k+1}\|^2} < \infty$$

$$\therefore \lim_{k\to\infty} inf \ \|g_k\| = 0$$

Proof:

$B_k^{MR} = (1-\theta_k)B_k^{Ls} + \theta_k B_k^{Ha}$

$B_k^{MR} = (1-\theta_k)\frac{g_{k+1}^T\, y_k}{-d_k^T\, g_k}\, d_k + \theta_k\, \frac{\|g_{k+1}\|^2}{(f_{k+1} - f_k/\alpha k - 3/2\,)d_k^T\, g_k} d_k$

$B_k^{MR} = \frac{g_{k+1}^T\ \ y_k}{-d_k^T\ \ g_k}\, d_k + \theta_k\, \frac{\|g_{k+1}\|^2}{(f_{k+1} - f_k/\alpha k - 3/2\,)d_k^T\, g_k} d_k$

$B_k^{MR} = \frac{\|y_k\|^2}{-d_k^T\ \ g_k} + \frac{\|g_{k+1}\|^2}{\delta_{\boldsymbol{\alpha k}}\, d_k^T\, g_k}$

$\therefore \|y_k\| = \|g_{k+1} - g_k\| \leq L\|S_k\|$

$$g_k^T d_k \leq -c\|g_k\|^2$$

$B_k^{MR} \leq \frac{L^2\|S_k\|^2}{c\|g_k\|^2} + \frac{\|g_{k+1}\|^2}{\delta_{\boldsymbol{\alpha k}}\, d_k^T\, g_k}$

$B_k^{MR} \leq \frac{L^2\|S_k\|^2}{c\|g_k\|^2} + \frac{\|g_{k+1}\|^2}{\delta_{\boldsymbol{\alpha k}}\, c\|g_k\|^2}$

$B_k^{MR} \leq \frac{L^2\|S_k\|^2}{cn_1\|S_k\|^2} + \frac{n_1\|S_k\|}{n_2\|S_k\|^2}$

$B_k^{MR} \leq \frac{L^2}{cn_1} + \frac{n_1}{n_2\|S_k\|}$

$$d_{k+1} = -g_{k+1} - B_k^{MR} d_k$$

$$\|d_{k+1}\| = \|-g_{k+1} + B_k^{MR} d_k\| \leq \|g_{k+1}\| + |B_k^{MR}|.\|d_k\|$$

$$\|d_{k+1}\|^2 = \|g_{k+1}\|^2 + 2B_k^{MR}\|g_{k+1}\|\|d_k\| + (B_k^{MR})^2.\|d_k\|^2$$

$$\leq n_2\|S_k\| + 2\left[\frac{L^2\|S_k\|}{cn_1\|S_k\|} + \frac{n_1}{n_2\|S_k\|}\right] n_2^{y_2}\|S_k\|^{y_2}\frac{\|S_k\|}{|\boldsymbol{\delta_{\alpha k}}|} + [\frac{L^2\|S_k\|}{cn_1\|S_k\|} + \frac{n_1}{n_2\|S_k\|}]^2\frac{\|S_k\|^2}{|\boldsymbol{\delta_{\alpha k}}|}$$

But

$$\|S_k\| = \|x_{k+1} - x_k\|$$

$$D = Max\,\{\|x_{k+1} - x_k\|, \forall\, x, x_k \in S\}$$

$$\leq n_2 D + 2\left[\frac{L^2 D}{cn_1} + \frac{n_1}{n_2}\right] n_2^{y_2}\frac{D^{y_2}}{|\delta_{\boldsymbol{\alpha k}}|} + \left[\frac{L^2 D}{cn_1} + \frac{n_1}{n_2}\right]^2 \cdot \frac{1}{|\delta_{\boldsymbol{\alpha k}}|^2}$$

$$\therefore \|d_{k+1}\|^2 \leq \psi$$

$$\sum_{k\geq 1}\frac{1}{\|d_{k+1}\|^2} \geq \sum_{k\geq 1}\frac{1}{\psi} = \frac{1}{\psi}\sum 1 = \infty \qquad (9)$$

$$\therefore \lim_{k\to\infty} inf\ \|g_k\| = 0.$$

5-Numerical Tests

The following part will analyse the results of our numerical experiments that employed the hybrid MR method. In addition, we will compare these results with the numerical outputs of two other algorithms (Hanen, LS) that utilise the Wolfe line search method. The comparison will be conducted using the metrics of iteration count (NoI) and function evaluation count (NoF). The iterations will stop when the magnitude of the gradient norm is less than or equal to 10^{-6}.

Furthermore, we utilized 30 unconstrained optimization problem functions with a variable count of either 100 or 1000. All the graphs in this work were generated using Fortran software. The discussion the results by Dolan and More miner to compare based on MATLAB program.

Table 1: The details of testing results using 30 functions with different dimension.

Function	Dim	Hanen		LS		MR	
		Ni	NF	Ni	NF	Ni	NF
Extended Trigonometric	100	37	111	32	73	26	45
	1000	79	228	90	661	41	70
Extended Rosenbrock SROSENBR	100	1001	1727	1001	1888	50	96
	1000	1001	1629	1001	2093	82	334
Extended White & Holst	100	1001	2090	1001	1712	68	195

	1000	1001	1777	1001	1586	56	121
Extended Beale BEALE (CUTE)	100	20	40	35	352	15	30
	1000	323	9893	160	4269	65	1462
Extended Penalty	100	268	1289	229	1134	154	374
	1000	1001	4322	1001	3975	577	1605
Perturbed Quadratic	100	258	987	206	581	136	254
	1000	1001	2136	713	2006	579	1138
Raydan 1	100	4	9	4	9	4	9
	1000	4	9	4	9	4	9
Generalized Tridiagonal 1	100	47	234	63	875	16	27
	1000	105	2582	135	3530	45	794
Extended Tridiagonal 1	100	105	477	85	386	50	133
	1000	199	1575	149	957	92	174
Extended Three Expo Terms	100	4	9	4	9	4	9
	1000	4	9	4	9	4	9
Generalized Tridiagonal 2	100	24	63	36	235	17	30
	1000	28	159	47	193	19	31
Diagonal 5	100	31	532	29	427	17	33
	1000	28	504	764	36	18	99
Generalized PSC1	100	56	91	57	93	16	30
	1000	65	105	65	105	65	105
Extended PSC1	100	1001	1481	1001	1615	72	161
	1000	1001	1351	1001	1313	125	309
Extended Block-Diagonal BD1	100	178	5279	64	1514	15	34
	1000	1001	33319	18	48	752	24585
Extended Maratos	100	268	621	212	515	148	270

	1000	1001	1583	100	2158	626	1183
Extended Cliff CLIFF (CUTE)	100	2	5	2	5	2	5
	1000	2	5	2	5	2	5
Quadratic Diagonal Perturbed	100	151	1472	187	2489	58	104
	1000	170	2894	113	1596	48	88
Quadratic QF1	100	121	613	99	419	22	108
	1000	514	1699	413	1978	9	56
Extended Quadratic Penalty QP1	100	1001	2487	1001	1935	19	38
	1000	1001	1273	1001	1621	31	31
Extended EP1	100	7	14	7	14	7	14
	1000	13	26	14	38	11	22
Extended Tridiagonal 2	100	10	18	10	18	10	18
	1000	11	19	11	19	11	19
BDQRTIC (CUTE)	100	13	24	13	24	13	24
	1000	14	25	14	25	14	25
TRIDIA (CUTE)	100	531	1658	594	1664	190	322
	1000	1001	2497	1001	2497	1001	1048
ARWHEAD (CUTE)	100	67	323	75	282	34	55
	1000	89	357	90	351	46	85
NONDQUAR (CUTE)	100	56	136	70	745	39	76
	1000	125	2585	66	513	44	378
DQDRTIC (CUTE)	100	11	29	11	29	11	29
	1000	16	43	16	43	16	43
DIXMAANA (CUTE)	100	478	1671	216	638	21	40
	1000	1001	1649	1001	1680	25	55
DIXMAANB (CUTE)	100	4	9	4	9	4	9
	1000	4	9	4	9	4	9
DIXMAANE (CUTE)	100	63	308	102	452	27	52

	1000	73	301	71	320	24	50
Partial Perturbed Quadratic	100	19	43	19	86	16	29
	1000	22	89	18	79	17	30
Tridiagonal Perturbed Quadratic	100	14	24	78	318	27	50
	1000	53	233	101	587	34	66
LIARWHD (CUTE)	100	31	532	29	427	17	33
	1000	28	504	35	764	39	76
NONDQUAR (CUTE)	100	56	136	70	745	44	378
	1000	125	2585	66	513	11	29
Total		16.809	12.988.506	16.820	306.020	5.833	36.873

We conclude the practical aspect by reviewing the graphics related to the policy of comparisons by researchers Dolan and More. The first drawing showed the differences between the new algorithm and the classic algorithms regarding the number of iterations. The second drawing was for standard function calculations.

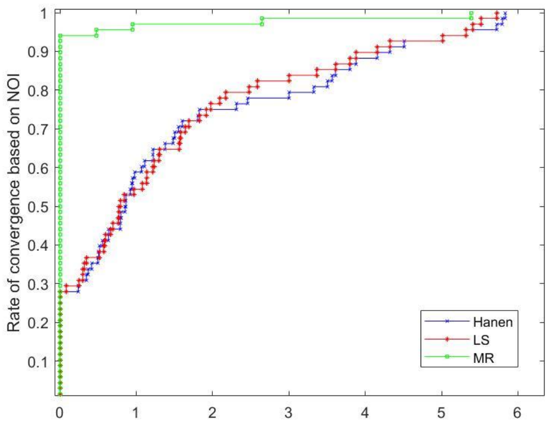

Figure 1: NoI comparsion.

We have observed that our strategy has transitioned to an ascending trajectory in contrast to the previous method.

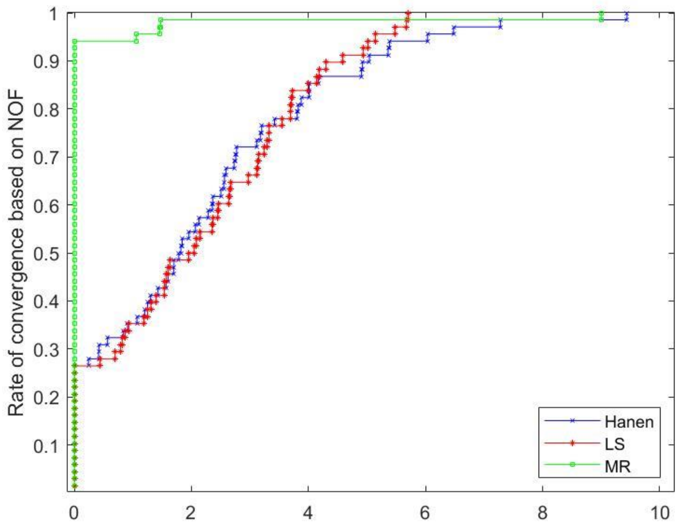

Figure 1: NoF comparison.

6-Conclusion

In this study, a novel strategy called β_k^H was created for unconstrained optimisation. It combines the algorithms β_k^{LS} and β_k^{Hanen} to create a hybrid conjugate gradient method. The suggested method has undergone thorough theoretical and practical analysis. The algorithm's features of adequate descent and global convergence have been confirmed by applying certain hypotheses.

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